BETWEEN EVENTS AND WORLDS UNDER HISTORICAL NECESSITY

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Abstract. Events and worlds under historical necessity are re-analyzed as *schedules* of eventuality-types. The perfect and non-root modals in Condoravdi 2002 are reformulated, reproducing over schedules the backward time shift in the perfect and the forward time expansion in modals.

1. Introduction

Just as (semantic) accounts of modality commonly invoke possible worlds, theories of temporality (eg aspect) often appeal to event[ualitie]s. But what are events? And what are worlds? We can investigate these questions together in works that combine events with worlds, such as Condoravdi 2002, henceforth C2. The perfect and non-root (= epistemic or metaphysical) modals are analyzed in C2 subject to *historical necessity* (Thomason 1984). That analysis is recounted briefly in §2. A notion of *schedule* that subsumes events and worlds is presented in §3, on the basis of which the perfect and the modals are reformulated in §4.

To more easily digest the formal details that follow, one of the many linguistic examples in C2 should help.

(1) He might have won.

We can read (1) epistemically as in (2) or metaphysically as in (3).

(2) He might have, for all I know, won.

(3) He might have, at that stage, won, but he didn't.

The words 'might' and 'have' are analyzed in C2 by functions MIGHT and PERF, in terms of which (1)'s epistemic reading arises from, roughly put,

$$\text{MIGHT}(\text{PERF}(\text{he-win}))$$

and (1)'s counterfactual reading from the reversed scoping

$$\text{PERF}(\text{MIGHT}(\text{he-win}))\ .$$

We spell this out more precisely next.

2. Temporal properties in C2

The semantic set-up in C2 assumes that the following are given:

(i) a set Pt of *temporal points/instants* linearly ordered by $\prec$, relative to which the set Ti of *times* can be conceived as consisting of non-empty subsets t of Pt (including singletons) such that for every $z \in \mathsf{Pt}$,

$$z \in t \quad \text{whenever } x \prec z \prec y \text{ for some } x, y \in t$$

(that is, time is a non-empty $\prec$-interval that may be open, closed or half-open)

(ii) sets Wo, Ev and St of *worlds*, of *events* and of *states*, respectively, along with a function $\tau : (\mathsf{Ev} \cup \mathsf{St}) \times \mathsf{Wo} \to (\mathsf{Ti} \cup \{\emptyset\})$ that specifies the *temporal trace* $\tau(e, w) \in \mathsf{Ti} \cup \{\emptyset\}$ *of* an event or state e *in* world w, where

$$\tau(e, w) = \emptyset \quad \text{iff} \quad e \text{ is not realized in } w \ ,$$

the intuition behind $\tau(e, w) \in \mathsf{Ti}$ being that e is a single token/occurrence in w (as opposed to a type that recurs in w).

An important difference between events and states appears when specifying, for all worlds w and times t, the events and states that (might be said to) *hold at* w, t. These can be collected in the sets

$$\begin{aligned} \mathsf{Ev}(w, t) &= \{e \in \mathsf{Ev} \ : \ \emptyset \neq \tau(e, w) \subseteq t\} \\ \mathsf{St}(w, t) &= \{e \in \mathsf{St} \ : \ \tau(e, w) \cap t \neq \emptyset\} \ . \end{aligned}$$

The idea is that at a fixed world w, a state e holds at any non-empty part of $\tau(e, w)$, whereas an event holds at t only if its entire temporal trace is contained in t. (Or, viewed from *outside* t, events are bounded while states are not.) To cash this out formally, C2 calls a function P from worlds

(i) *eventive* if for every world w, $P(w)$ is a unary predicate on events (so $P(w)(e)$ is either true or false for every event e)

(ii) *stative* if for every world w, $P(w)$ is a unary predicate on states (so $P(w)(e)$ is either true or false for every state e)

(iii) *temporal* if for every world w, $P(w)$ is a unary predicate on times (so $P(w)(t)$ is either true or false for every time t)

(iv) a *property* if P is eventive or stative or temporal

and defines a way to turn any property P to a temporal property — viz $\lambda w \lambda t \ \mathrm{AT}(t, w, P)$, where

$$\mathrm{AT}(t, w, P) \ = \ \begin{cases} (\exists e \in \mathsf{Ev}(w, t)) \ P(w)(e) & \text{if } P \text{ is eventive} \\ (\exists e \in \mathsf{St}(w, t)) \ P(w)(e) & \text{if } P \text{ is stative} \\ P(w)(t) & \text{if } P \text{ is temporal.} \end{cases}$$

AT is used to formalize both the perfect and the modals. A function PERF mapping properties P to temporal properties is defined by

$$(\mathrm{PERF}\ P)(w)(t) \ = \ (\exists t' \prec t) \ \mathrm{AT}(t', w, P)$$

where the linear order $\prec$ on Pt is extended to Ti according to

$$t \prec t' \quad \text{iff} \quad (\forall x \in t)(\forall x' \in t')\ x \prec x'.$$

To analyze modals, a *modal base* function MB is assumed that maps a world-time pair (w, t) to a set of worlds, relative to which a function MIGHT_{MB} maps a property P to the temporal property satisfying

$$(\text{MIGHT}_{\text{MB}}\ P)(w)(t) \quad = \quad (\exists w' \in \text{MB}(w,t))\ \text{AT}(t_\infty, w', P)$$

where (expanding time forward, as in Abusch 1998) t_∞ is the interval

$$\{x \in \mathsf{Ti} \ :\ (\exists y \in t)\ y \preceq x\}$$

(with $y \preceq x$ abbreviating '$y \prec x$ or $x = y$'). One of the main points of C2 is that the modal base MB in the "present perspective"

(4) $\quad (\text{MIGHT}_{\text{MB}}(\text{PERF}\ P))(w)(t) \ = \ (\exists w' \in \text{MB}(w,t))(\exists t' \prec t)\ \text{AT}(t', w', P)$

is epistemic, while that in the "past perspective"

(5) $\quad (\text{PERF}(\text{MIGHT}_{\text{MB}}\ P))(w)(t) \ = \ (\exists t' \prec t)(\exists w' \in \text{MB}(w,t'))\ \text{AT}(t'_\infty, w', P)$

is metaphysical.

The contrasting modal bases in (4) and (5) is traced in C2 to historical necessity, according to which (metaphysically) the past is completely settled, and only the future is open to branching. This view can be formalized by bundling together at every time t, worlds that are identical on all previous times $t' \prec t$. This bundling is effected through a a family $\{\simeq_t\}_{t \in \mathsf{Ti}}$ of equivalence relations $\simeq_t$ on Wo indexed by times t such that

(H1) $\quad$ whenever $w \simeq_t w'$ and $t' \prec t$, $\ w \simeq_{t'} w'$

and

(H2) $\quad$ for every temporal property $\hat{P}$ *of interest*,

$$\hat{P}(w)(t) \quad \text{iff} \quad (\forall w' \simeq_t w)\ \hat{P}(w')(t)\ .$$

The qualification "of interest" in (H2) is necessary to allow for branching in the future; otherwise, $\simeq_t$'s satisfying (H2) must be equality, in view of *un*interesting temporal properties such as those given, for every world w, by

$$(\forall w' \in \mathsf{Wo})(\forall t \in \mathsf{Ti}) \qquad \hat{P}(w')(t) \ \text{ iff } \ w' = w\ .$$

To rule out a metaphysical reading of (4), it suffices that we count among the $\hat{P}$'s the temporal predicate $\lambda w \lambda t\ \text{AT}(t, w, P)$, for P in (4). The argument then comes down to the assumption that for all worlds w and times t,

(6) $\quad (\forall w' \in \text{MB}(w,t))\ \ w \simeq_t w'$

provided MB is a metaphysical modal base, in which case (4) reduces to $(\text{PERF}\ P)(w,t)$. By contrast, in (5), a metaphysical modal base need not render MIGHT_{MB} superfluous. And as for epistemic readings, condition (6) fails (unless everything about the past is known).

3. Scheduling eventive and stative properties

To investigate what a world might be, let us see just what it contributes to semantic interpretation. Toward that end, let EP be a set of symbols for eventive and stative properties, and let Φ be a set of formulas that includes EP ($\subseteq \Phi$). An interpretation of Φ relative to world-time pairs can be encoded as a relation $\models \subseteq (\mathsf{Wo} \times \mathsf{Ti}) \times \Phi$. We might define '$w, t \models \dot{P}$' as in the previous section according to (7).

(7) $\quad w, t \models \dot{P} \quad$ iff $\quad \mathrm{AT}(t, w, P)$

An obvious alternative to (7) is (8).

(8) $\quad w, t \models \dot{P} \quad$ iff $\quad \exists e[P(w)(e) \ \&\ \tau(e, w) = t]$

(7) weakens the equality $\tau(e, w) = t$ in (8) to $\tau(e, w) \subseteq t$ in case P is eventive, and to $\tau(e, w) \cap t \neq \emptyset$ in case P is stative. With this slack, it follows that eventive and stative properties P which hold at t hold at any $t' \supseteq t$

$$\mathrm{AT}(t, w, P) \text{ and } t \subseteq t' \quad \text{imply} \quad \mathrm{AT}(t', w, P) \ .$$

A case of special interest is the time expansion $t_\infty \supseteq t$ in MIGHT, on which C2's claim that "there is no tense in the scope of the modal" rests. In particular, PERF does not, in the scheme of C2, operate on tense (in natural language). Afterall, PERF is based on a predicate, AT, that is sensitive to the difference between events and states. On temporal properties P, however,

$$(\mathrm{PERF}\ P)(w)(t) \quad = \quad (\exists t' \prec t)\ P(w)(t')$$

and it is difficult to discern any divergence from the Priorian past tense operator. An attempt will be made in the next section to show that the concerns of C2 are, in fact, better served by (8).

Whichever of (7) or (8) is adopted, it will prove useful to pass from a world w to the function $s_w : \mathsf{Ti} \rightarrow \mathit{Power}(\mathsf{EP})$ given by

$$s_w(t) \quad = \quad \{\dot{P} \in \mathsf{EP} : \ w, t \models \dot{P}\}$$

for $t \in \mathsf{Ti}$. We can then define for every $\hat{t} \in \mathsf{Ti}$, an equivalence relation $\simeq_{\hat{t}} \subseteq \mathsf{Wo} \times \mathsf{Wo}$ by

$$w \ \simeq_{\hat{t}} \ w' \quad \text{iff} \quad (\forall t \preceq \hat{t})\ s_w(t) = s_{w'}(t) \ .$$

Conditions (H1) and (H2) for historical necessity are met, at least if the temporal properties "of interest" in (H2) are restricted to those with symbols in EP. Focussing on functions from Ti to $\mathit{Power}(\mathsf{EP})$, let us define for all $s, s' : \mathsf{Ti} \rightarrow \mathit{Power}(\mathsf{EP})$ and $\hat{t} \in \mathsf{Ti}$,

$$s\ R_{\hat{t}}\ s' \quad \text{iff} \quad (\forall t \preceq \hat{t})\ s(t) = s'(t) \ .$$

If we identify a world w with s_w, then condition (6) from the previous section yields

$$\mathrm{MB}(s, t) \ \subseteq \ \{s' : s\ R_t\ s'\} \ .$$

We shall see in the next section how to do away with the pesky time subscript t in R_t, and get the time expansion t_∞ in MIGHT to drop out. The key is to generalize from (total) functions (from Ti to $Power(\mathsf{EP})$) to *partial* functions. With that in mind, let us define a *schedule* to be a partial function from Ti to $Power(\mathsf{EP})$. The schedule s_w induced by w can be constructed from schedules $s_{e,w}$ induced by eventualities e that are realized in w (i.e. $\tau(e,w) \neq \emptyset$). Assuming (8), we need only set for such e's,

$$dom(s_{e,w}) = \{\tau(e,w)\} \quad \text{and} \quad s_{e,w}(\tau(e,w)) = \{\dot{P} \in \mathsf{EP} : P(w)(e)\}$$

in order to establish (9) for all $t \in \mathsf{Ti}$.

(9) $\quad s_w(t) \;=\; \bigcup\{s_{e,w}(t) \;:\; e \in \mathsf{Ev} \cup \mathsf{St} \text{ and } \tau(e,w) = t\}$

It is possible also under (7) to derive (9), assuming a natural definition of $s_{e,w}$ that I leave to the interested reader.

4. Sharpening the perfect and the modals

The backward shift '$\exists t' \prec t$' effected by (PERF P)$(w)(t)$ can be recreated in a version pErf of PERF that replaces time t by an eventuality (= state or event) e, with

$$(\text{pErf } P)(w)(e) \quad \text{iff} \quad \exists e'[e' \supset\!\subset e \;\&\; P(w)(e') \;\&\; P_{cs}(w)(e)]$$

where

(i) '$e' \supset\!\subset e$' is the abutment condition in (for example) Kamp and Reyle 1993 stating that e' is temporally located immediately prior to e

and

(ii) P_{cs} is some contextually determined stative property that (notably) Steedman has argued constrains the consequent state (e'') of a P-eventuality (e').

PERF effectively chooses P_{cs} to be vacuous, with $P_{cs}(w)(e)$ true for all worlds w and states e. In general, abutment corresponds to a successor relation *succ* on times t, t'

$$succ(t',t) \quad \text{iff} \quad t' \prec t \;\&\; \neg(\exists t'' \in \mathsf{Ti})(t' \prec t'' \prec t)$$

in a transcription of pErf over to $\models\; \subseteq (\mathsf{Wo} \times \mathsf{Ti}) \times \Phi$

$$w,t \models \text{perf } \varphi \quad \text{iff} \quad w,t \models \varphi_{cs} \text{ and } \exists t'[succ(t',t) \;\&\; w,t' \models \varphi]$$

(where, for an inductive construal, φ_{cs} must be less $\models$-complex than φ). Applying AT to define '$w,t \models \dot{P}$' according to (7) would fail to insure that a consequent state comes immediately after the eventuality of which it is a consequence, defeating the tightness that *succ* adds to $\prec$. In this sense, pErf/perf is a finer instrument than PERF. But can it do the job on modals that PERF does?

To show that it can, it will be convenient to combine world-time pairs w,t (used by $\models$ to interpret Φ) into a schedule $s_{w,t}$ obtained by restricting s_w to times that are in a precise sense no later than t

$$s_{w,t} \;=\; \{(t',\alpha) \in s_w \;:\; t' \text{ is no later than } t\}\ .$$

An obvious candidate for "no later than" is the reflexive closure $\preceq$ of $\prec$

$$t' \preceq t \quad \text{iff} \quad t' = t \text{ or } (\forall x' \in t')(\forall x \in t)\ x' \prec x\ .$$

The problem is that for the applications below to MIGHT, $\preceq$ is too restrictive; for instance, $[0,1] \not\preceq [0,2]$ under the usual ordering of real numbers. Let us choose instead the pre-order $\preceq_\circ$ on Ti defined by

$$t' \preceq_\circ t \quad \text{iff} \quad t' \preceq_l t\ \&\ t' \preceq_r t$$

where $\preceq_l$ insures that the left boundary of t' is no later than that of t

$$t' \preceq_l t \quad \text{iff} \quad (\forall x \in t)(\exists x' \in t')\ x' \preceq x$$
$$[\ \text{iff} \quad t \subseteq t'_\infty\ ]$$

while $\preceq_r$ attends to the right boundaries

$$t' \preceq_r t \quad \text{iff} \quad (\forall x' \in t')(\exists x \in t)\ x' \preceq x\ .$$

That is, over $\prec$-intervals $[\mathit{left}', \mathit{right}']$ and $[\mathit{left}, \mathit{right}]$,

$$[\mathit{left}', \mathit{right}'] \preceq_\circ [\mathit{left}, \mathit{right}] \quad \text{iff} \quad \mathit{left}' \preceq \mathit{left}\ \&\ \mathit{right}' \preceq \mathit{right}\ .$$

(The pre-orders $\preceq_l$, $\preceq_r$ and $\preceq_\circ$ are known as the *Smyth, Hoare* and *Plotkin pre-orders induced by* $\prec$, respectively; Plotkin 1981.) Clearly,

$$t' \preceq t \quad \text{implies} \quad t' \preceq_\circ t\ .$$

Now, let us call a schedule s a *sked* if $s = s_t$ for some $t \in \mathit{dom}(s)$, where s_t is the restriction of s to times $\preceq_\circ t$

$$s_t \quad = \quad \{(t', \alpha) \in s : t' \preceq_\circ t\}\ .$$

It is easy to see that if $s = s_t$ for some $t \in \mathit{dom}(s)$, then that t must be

$$\bigcap \quad \{t' \in \mathit{dom}(s) : (\forall t'' \in \mathit{dom}(s))\ t'' \preceq_r t'\}$$

which we henceforth denote $\mathsf{last}(s)$ — intuitively, the least, latest time in $\mathit{dom}(s)$.

Next, having truncated schedules to skeds, let us replace the world-time pairs (w, t) in $\models$ by skeds $s_{w,t}$. Given a set Σ of skeds, we repackage $\models$ into a relation $\models_\Sigma \subseteq \Sigma \times \Phi$, where for all $s \in \Sigma$, $\dot{P} \in \mathsf{EP}$ and $\varphi \in \Phi$,

$$s \models_\Sigma \dot{P} \quad \text{iff} \quad \dot{P} \in s(\mathsf{last}(s))$$
$$s \models_\Sigma \text{Perf}\ \varphi \quad \text{iff} \quad s \models_\Sigma \varphi_{cs}\ \&\ \exists s'[s' \subseteq s\ \&\ \mathit{succ}(\mathsf{last}(s'), \mathsf{last}(s))\ \&\ s' \models_\Sigma \varphi]\ .$$

As for MIGHT, the idea roughly is to use, for a metaphysical reading, the modal base

$$\text{MB}(s) \quad = \quad \{s' : s\ R_{\mathsf{last}(s)}\ s'\}$$

where R_t is as defined back in §3. More precisely, let $\sqsubseteq_\circ$ be the binary relation between skeds s, s' such that

$$s \sqsubseteq_\circ s' \quad \text{iff} \quad s' = s \cup \{(t, \alpha) \in s' : t \not\preceq_\circ \mathsf{last}(s)\}\ .$$

That is,

$$s \sqsubseteq_\circ s' \quad \text{iff} \quad s \subseteq s'\ \&\ (\forall t \in \mathit{dom}(s') - \mathit{dom}(s))\ t \not\preceq_\circ \mathsf{last}(s)\ .$$

Let us say s' is an *end extension of* s to mean $s \sqsubseteq_\circ s'$, which we shall also write $s' \sqsupseteq_\circ s$. An end extension s' of s shifts neither the left nor right boundaries of $\mathsf{last}(s)$ backward.

(10) $s \sqsubseteq_\circ s'$ implies $\mathsf{last}(s) \preceq_\circ \mathsf{last}(s')$.

For a metaphysical reading $\langle \mathsf{meta} \rangle \varphi$ of *might* φ, let

$$s \models_\Sigma \langle \mathsf{meta} \rangle \varphi \quad \text{iff} \quad (\exists s' \sqsupseteq_\circ s)\ s' \models_\Sigma \varphi$$

for $s \in \Sigma$. How do we choose an end extension s' of s (indicated above by '$\exists s' \sqsupseteq_\circ s$') to match the effect in MIGHT of AT and the temporal expansion $\mathsf{last}(s)_\infty$? This is trivial if AT is built into $\models$ as in (7), since $\mathsf{last}(s) \preceq_\circ \mathsf{last}(s)_\infty$. But what about (8), for which there is no predicate AT on which to shove the search/existential quantifier? Under (8), s' must be chosen so that $\mathsf{last}(s')$ coincides precisely with the temporal trace $\tau(e, w)$ that in MIGHT, AT locates relative to $\mathsf{last}(s)_\infty$. Assuming

(†) such a choice can be made for eventualities e for which $\tau(e, w) \subseteq \mathsf{last}(s)_\infty$

there is still the case of states e for which $\tau(e, w) \neq \tau(e, w) \cap \mathsf{last}(s)_\infty \neq \emptyset$. But for such states e, it suffices that there be some state e' identical to e except that

$$\tau(e', w) \ = \ \tau(e, w) \cap \mathsf{last}(s)_\infty \quad (\subseteq \mathsf{last}(s)_\infty)$$

for then we can appeal to (†). The proliferation of states required here is, I think, a small price to pay to eliminate AT. Afterall, it is the divisible/point-like character of states, as opposed to the interval-like nature of events, that lies behind the different treatment AT accords to states and events. This brings us back to the assumption (†) that any event or state e such that $\tau(e, w) \subseteq \mathsf{last}(s)_\infty$ can be captured by an end extension s' of s. In fact, we cannot under the present set-up count on (†), but only (‡)

(‡) for any eventuality e such that $\mathsf{last}(s) \preceq_\circ \tau(e, w)$, there is an end extension s' of s such that $\mathsf{last}(s') = \tau(e, w)$.

Under (‡), end extensions of s need only zero in on eventualities e that shift the right endpoint of $\mathsf{last}(s)$ forward

$$\mathsf{last}(s) \ \preceq_r \ \tau(e, w) \ ,$$

in addition to the left

$$\mathsf{last}(s) \ \preceq_l \ \tau(e, w)$$

(the latter being $\tau(e, w) \subseteq \mathsf{last}(s)_\infty$). I am not aware of any arguments for retracting the right endpoint of $\mathsf{last}(s)$, the very idea of which runs counter that of possibilities unfolding as time marches inexorably forward. Given that the left endpoint of $\mathsf{last}(s)$ cannot move to the left, one can only pull the right endpoint of $\mathsf{last}(s)$ back so far before the interval becomes empty. In particular, if $\mathsf{last}(s)$ is so small as to be point-like, then there may not be any room left for its right endpoint. This is the case for the explanation given in C2 for the contrast between the optional forward shift of modals on states (illustrated by (11)) and the obligatory forward shift for events (as in (12)).

(11) She might be here (right now/tomorrow).

(12) She might talk.

Whereas a point-like state in (11) can fit into $\mathsf{last}(s)$, a talk-interval for (12) would not. Leaving out the modal and concentrating on 'now', there is pressure to pass from an interval-like event to a point-like state in (13).

(13) A man [$^?$walks/is walking] in the park.

Turning to an epistemic reading of MIGHT, and recalling (10), let us agree that for $s \in \Sigma$,

$$s \models_\Sigma \langle \mathsf{epis} \rangle \varphi \quad \text{iff} \quad \exists s' [\mathsf{last}(s) \preceq_\circ \mathsf{last}(s') \ \& \ s' \models_\Sigma \varphi] \ .$$

If the set Σ (relative to which $\models$ is defined) is understood as the *common ground*, it is natural to define a dynamic/context-change interpretation where context $= \Sigma$. Asserting φ at t updates Σ to

$$\Sigma_{\varphi,t} \quad = \quad \{s \in \Sigma : (\exists s' \in \mathsf{compatible}_\Sigma(s))(\mathsf{last}(s') = t \ \& \ s' \models_\Sigma \varphi)\}$$

where $\mathsf{compatible}_\Sigma(s)$ is the subset of Σ compatible with s

$$\mathsf{compatible}_\Sigma(s) \quad = \quad \{s' \in \Sigma : (\exists s'' \in \Sigma)(s \sqsubseteq_\circ s'' \ \& \ s' \sqsubseteq_\circ s'')\} \ .$$

Information grows by narrowing the range Σ of possibilities

$$\Sigma_{\varphi,t} \ \subseteq \ \Sigma \ .$$

The reader familiar with Veltman 1996 may note that $\langle \mathsf{epis} \rangle \varphi$ (but not $\langle \mathsf{meta} \rangle \varphi$) is a test in that

$$\Sigma_{\langle \mathsf{epis} \rangle \varphi, t} = \Sigma \quad \text{or} \quad \Sigma_{\langle \mathsf{epis} \rangle \varphi, t} = \emptyset$$

under natural assumptions on Σ. This can be put down to the fact that for all $s, s' \in \Sigma$ such that $\mathsf{last}(s) = \mathsf{last}(s')$,

$$s \models_\Sigma \langle \mathsf{epis} \rangle \varphi \quad \text{iff} \quad s' \models_\Sigma \langle \mathsf{epis} \rangle \varphi \ .$$

5. Conclusion

The thrust of the present work is to treat events and worlds uniformly as schedules of eventuality-types, where the set of eventuality-types is written EP above, and a schedule is a partial function from times to subsets of EP. Times are conceived as intervals in order to accommodate events and the time extension t_∞ that C2 builds into the semantics of modalities. A pre-order $\preceq_\circ$ is defined on times, relative to which schedules are truncated into skeds, and skeds are related by end-extensions/$\sqsubseteq_\circ$. Under the structure provided by $\sqsubseteq_\circ$, a sked is a schedule that for some time t, essentially associates with every time $t' \preceq_\circ t$ what McCarthy and Hayes 1969 call a *situation*, "a complete state of the universe at" t'. The assumption here of completeness for all times $t' \preceq t$ is crucial for the analysis above of modalities under historical necessity.

Stepping beyond worlds and modalities, over to events, the relation $\sqsubseteq_\circ$ generalizes to a partial order $\sqsubseteq$ on schedules defined by

$$s \sqsubseteq s' \quad \text{iff} \quad dom(s) \subseteq dom(s') \text{ and } (\forall t \in dom(s)) \ s(t) \subseteq s'(t)$$

where '$s \sqsubseteq s'$' is read: s has information content less than or equal to that of s'. This partial order is in line with equation (9) in §3. Indeed, under $\sqsubseteq$, schedules can be viewed as *forcing conditions* in the sense of Keisler 1973, with worlds as *generic sets*, provided a suitable notion of consistency is defined on the (power)set of eventuality-types. (Details in Fernando 2002, where $\sqsubseteq$ links worlds to events, as conceived in, for example, Moens and Steedman 1988 and Steedman 2000.) To get at such a notion of consistency, an attempt is made above to eliminate the use in C2 of AT, allowing, as it does, mutually inconsistent eventuality-types to hold within the same interval. That is, for all worlds w, times t, eventualities e, e' and eventuality-properties P, P',

$$P(w)(e) \ \& \ P'(w)(e') \ \& \ t \supseteq \tau(e,w) \cup \tau(e',w) \quad \text{imply} \quad \mathrm{AT}(t,w,P) \ \& \ \mathrm{AT}(t,w,P') \ .$$

The two uses of $\exists$ in

$$(\mathrm{MIGHT}_{\mathrm{MB}} \ P)(w)(t) \quad = \quad (\exists w' \in \mathrm{MB}(w,t)) \ \mathrm{AT}(t_\infty, w', P) \ ,$$

one binding w' and another buried in AT, are merged in

$$s \models_\Sigma \langle \mathsf{meta} \rangle \varphi \quad \text{iff} \quad (\exists s' \sqsupseteq_\circ s) \ s' \models_\Sigma \varphi \ ,$$

refining the time expansion t_∞. But before putting AT away, let us consider a universal variant of MIGHT, called WOLL, that C2 defines as

$$(\mathrm{WOLL}_{\mathrm{MB}} \ P)(w)(t) \quad = \quad (\forall w' \in \mathrm{MB}(w,t)) \ \mathrm{AT}(t_\infty, w', P) \ .$$

The universal forms

$$\begin{aligned} s \models_\Sigma [\mathsf{meta}]\varphi \quad &\text{iff} \quad (\forall s' \in \Sigma)[s \sqsubseteq_\circ s' \text{ implies } s' \models_\Sigma \varphi] \\ s \models_\Sigma [\mathsf{epis}]\varphi \quad &\text{iff} \quad (\forall s' \in \Sigma)[\mathsf{last}(s) \preceq_\circ \mathsf{last}(s') \text{ implies } s' \models_\Sigma \varphi] \end{aligned}$$

fail to capture the $\forall\exists$ complexity that AT gives to WOLL. Nor is it enough to translate φ to $\langle \mathsf{meta} \rangle \varphi$ before applying [meta] and [epis]

$$\begin{aligned} s \models_\Sigma [\mathsf{meta}]\langle \mathsf{meta} \rangle\varphi \quad &\text{iff} \quad (\forall s' \in \Sigma)[s \sqsubseteq_\circ s' \text{ implies } (\exists s'' \sqsupseteq_\circ s') \ s'' \models_\Sigma \varphi] \\ s \models_\Sigma [\mathsf{epis}]\langle \mathsf{meta} \rangle\varphi \quad &\text{iff} \quad (\forall s' \in \Sigma)[\mathsf{last}(s) \preceq_\circ \mathsf{last}(s') \text{ implies } (\exists s'' \sqsupseteq_\circ s') \ s'' \models_\Sigma \varphi] \end{aligned}$$

as s'' may have to be chosen between s and s'. That is, we must reject the simple forms $[\mathsf{meta}]\varphi$ and $[\mathsf{epis}]\varphi$ in favor of

$$\begin{aligned} s \models_\Sigma [\mathsf{meta}]'\varphi \quad &\text{iff} \quad (\forall s' \in \Sigma)[s \sqsubseteq_\circ s' \text{ implies} \\ &\qquad\qquad (\exists s'' \sqsupseteq_\circ s)(\mathsf{compatible}_\Sigma(s', s'') \ \& \ s'' \models_\Sigma \varphi)] \\ s \models_\Sigma [\mathsf{epis}]'\varphi \quad &\text{iff} \quad (\forall s' \in \Sigma)[\mathsf{last}(s) \preceq_\circ \mathsf{last}(s') \text{ implies} \\ &\qquad\qquad (\exists s'' \sqsupseteq_\circ s)(\mathsf{compatible}_\Sigma(s', s'') \ \& \ s'' \models_\Sigma \varphi)] \end{aligned}$$

where the predicate $\mathsf{compatible}_\Sigma$ (defined for the context change interpretation in §4) matches s' up with s''. Merging quantification on worlds with time may no longer seem so appealing. For all the slack it introduces, there is something quite irresistible about how AT sorts worlds from times.

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