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# **New JLS-Factor Model: Deep Analysis on Bitcoin Bubble**

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DISSERTATION REPORT

IN COMPLETION OF THE DEGREE PROGRAM IN

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SCHOOL OF COMPUTER SCIENCE & STATISTICS

UNIVERSITY OF DUBLIN, TRINITY COLLEGE DUBLIN, IRELAND

SUPERVISOR: PROF. BAHMAN HONARI

SEPTEMBER 2020

# Declaration

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September 6, 2020

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### Abstract

In this paper, we present the New Johansen-Ledoit-Sornette (JLS) model by incorporating market fundamental economic factors such as historical volatility of an asset into old JLS Factor model on bitcoin daily price. The stock market assets esp. Bitcoin has been the focus of the study for many researchers since very long time as bitcoin is very volatile. Also, the bubble in any kind of asset is usually characterized by log-periodic power law (LPPL model) since such bubbles follow "faster-than-exponential" growth. This New JLS factor model is presented with a small transformation in which the equation for bubble and crash reduces to the function of linear and non-linear parameters. Since Bitcoin daily price has seen unprecedented growth since last 2 years, this New JLS factor model will try to predict Bitcoin Bubble and Crash in 2017 followed by another local bubble and crash in 2020. There are two time frames chosen in each year: September to December 2017 and December 2019 to February 2020 for bubble phase. We validate the results from the model by utilizing evolution Strategy esp. Covariance-Matrix Adaptation Evolution Strategy (CMA-ES), a gradient-free optimisation algorithm. The main focus is on the critical parameter called time to peak( $t_c$ ) by employing above methodology since this is when bubble price will take plunge and oscillation frequency will start going down. Evidences are given in form of non-linear and linear parameter values. Finally, this model makes price prediction of Bitcoin post August 10, 2020 by 60-day and 90-day forward rolling window and it is expected to see another local or global bubble by Mid-September, 2020. Based on the evidences, our model can be extended further to study other stocks as well.

**Keywords:** New JLS factor model, Bubbles, Crashes, log-periodic power law, CMS-ES, optimization, gradient-free and evolution strategy.

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# Chapter 1

## Introduction and Overview

### 1.1 Background

Bitcoin is a combination of technologies and different concepts to generate a digitalised money system [5]. The currency in the bitcoin flows from one participant to another in digital way inside the network. All communication happens via bitcoin protocol. This protocol can be run on multiple devices such as laptops, phones or other devices using software available as open source.

Customers can trade in bitcoin currency in a digital way. Bitcoin is termed as distributed, peer to peer system by Satoshi Nakamoto in 2008 in his paper [29]. The bitcoins are generated by a process called mining. Bitcoin has no central server for control. Anyone in the bitcoin network can behave as a miner and can perform mining operation. The transactions done by each user can be validated in each 10 minutes interval. The detailed explanation about Bitcoin network, protocol and mining process is presented in literature review section.

Bitcoin has gained popularity among investors since its price movement over the years and its increasing market capitalisation in last 10 years as given in below Figure 1.1. From the year 2017, participants started believing in bitcoin to be traded as a financial asset. There were a lot of market players and media reports involved to promote bitcoin as an asset. More details about bitcoin function as a financial asset are provided in literature review section. The price of bitcoin fluctuated between \$1000 to \$20,000 per coin in just a period of 10 months between year 2017 and 2018. Such a movement draws criticisms and customer focus towards it.

That is the main reason to perform deep analysis on bitcoin price movement over the years. Many economists have performed deep statistical analysis earlier on different kind of stocks. The deep analysis on bitcoin using statistical approach requires more understanding of the statistical equation since the bitcoin price movement does not follow any exponential pattern but faster than exponential pattern. We shall be conducting this deep analysis on bitcoin using New JLS factor model on different

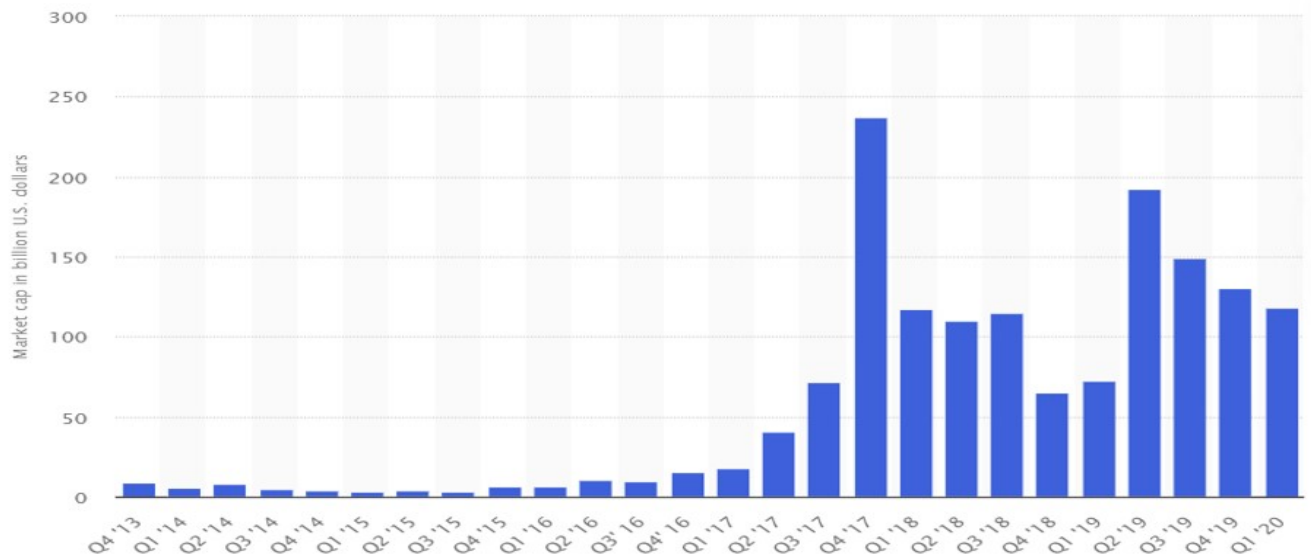


Figure 1.1: Bitcoin Market Capitalization

Source: <https://www.statista.com/statistics/377382/bitcoin-market-capitalization/>

time frames by introducing more parameters in order to match the exact bitcoin price movement.

## 1.2 Motivation

Earlier studies about Bitcoin have more focus on bitcoins' technical details and its usability in blockchain. There is an area which has not been focused much called financial asset. Also, Bitcoin price has seen unprecedented growth after 2014. Bitcoin market capitalization has touched 220 billion USD in 2017 and kept fluctuating between 2017 and 2020. Hence our focus is to study the bitcoin price movement over these years. In this thesis, we will focus on the bitcoin bubble and crash that happened in 2017 and 2020. Since bitcoin is a volatile asset and it does not follow classical financial theory [20]. We will be exploring statistical models such as JLS and New JLS factor model since these models focus on those stocks which follow faster than exponential growth. As per the Johansen-Ledoit-Sornette (JLS) model, the stock market bubble and crash are the result of imitation between participants and their price herding behaviour. The JLS model does not incorporate market fundamental factors such as deposit reserve rate, interest rate and historical volatility of the asset over a period when making prediction. Hence we will focus on the New Johansen-Ledoit-Sornette (JLS) model by introducing market-related factors in order to study the bitcoin bubble and its price movement [20].

I am a Data Science student and have keen interest in applying statistical modelling on financial data since I have worked on different types of data-sets earlier with my previous organizations and I have explored Artificial Intelligence on Trading data, too. By writing this thesis, I will also get benefited

by learning statistical models and by applying optimisation algorithm which I learnt in other modules.

### 1.3 Objectives

The main purpose of this thesis is to analyse the bubble and crash phase of Bitcoin between year 2017 and 2020 using New JLS factor model. Since Bitcoin had shown significant price change after 2014. We are more interested to study its behaviour in different time windows. In order to analyse bubble phase, we will pick time window in the year 2017 and then another time window of 60 days to predict crash phase. Similarly, two time windows will be picked in the year 2020 to predict another bubble and crash. Our dissertation also focuses on predicting bitcoin price in future for the month of September, 2020. Since the beginning of this year 2020, the price of bitcoin has dropped to \$4,994 in the month of March, 2020 and since then, it is increasing and has reached to \$12,000 per coin. Can we expect another bubble and then crash? We will use bitcoin price data upto August 10, 2020 in order to predict prices for the month of September 2020 which can be validated in future itself. However, other windows chosen for bubble and crash prediction can be verified using historical data already available from coin desk website and we will be plotting these prices in other sections of the thesis.

Another purpose of the study is to validate model parameters using evolution strategy. In New JLS factor model, there are almost 8 parameters to be estimated. This price movement is a non-convex optimization problem. Hence we will use Covariance Matrix Adaptation Evolution Strategy(CMA-ES), a gradient-free optimization problem [18].

Also, in addition to our main thesis objectives, we will be answering below questions in literature review.

- Key points to differentiate between Cryptocurrency and Stock Market.
- What is economic bubble and crash and the life cycle of a bubble?
- What key factors will be used to predict the near-real behaviour of the bitcoin price?

### 1.4 Structure

The entire thesis is structured as given below:

- Chapter 1 talks about background, motivation and objectives to perform this dissertation.
- Chapter 2 discusses more facts about cryptocurrency, its comparison with stock market, bubble, and crash. This chapter also mentions Bitcoin features and models used so far to study bitcoin

behaviour in the past.

- Chapter 3 is about our research why does this specific topic interest us the most?
- Chapter 4 talks about model description (New JLS factor model) and its equation derivation along with slight transformation for anti-bubble phase.
- Chapter 5 focuses on data collection and data analysis. This chapter also presents analysis and comparison of bitcoin price with other assets.
- Chapter 6 is all about implementation and validation phase of the model. This implementation was done for both the years 2017 and 2020. CMA-ES algorithm is used to validate the results and to reduce the error from the prediction.
- Chapter 7 produces results after implementation in the previous chapter.
- Chapter 8 talks about conclusion of this thesis overall.
- Chapter 9 mentions our model limitation and future scope to enhance the model.

## Chapter 2

# Literature Review

### 2.1 Comparison between Cryptocurrency and Stock Market

Before we start diving deep into bitcoin price movement, we shall be understanding differences between Cryptocurrency and stock market. Few facts are given below as follows.

1. Cryptocurrencies are of global nature and have a global attraction to all participants interested to invest while stocks or specific assets are dealt in same market where they belong to [22].

2. Banks, Security agencies and Governments are the regulators behind stock markets; however, cryptocurrency does not have any such regulations with absence of any kind of centralized governance. Bitcoin or any such cryptocurrency are not centralized though blockchain does provide ways to manage it. This is the main reason for customers' lack of trust in such market, causing higher price movement in a short span of time. According to Shehhi and Oudah (2014), it is challenging to attract investors for an asset which is not regulated by the government agencies rather than asset which are regulated [2].

3. Market factors and manipulations affect cryptocurrency more than they affect stock markets since stocks show reluctance behaviour. Cryptocurrency can be good assets to play with for those who love to surmise and bring emotions such as fear, doubtness and uncertainty in the market about the future. (Forbes, 2018).

4. There is a lot of extra charges such as brokerage, taxes, cost related to transaction and transfer fees while trading stocks; however, such charges do not exist in case of cryptocurrency, making it more attractive to individuals and causing more volatile market as per (Shehhi and Oudah, 2014) [2].

5. Major key difference between cryptocurrency and stock is the time to make profit and loss. Everyone wants to gain profit immediately after investing but it takes time when someone is trading in stock market; however, bitcoin can be profitable within short span of time as well as making loss for the same reason as per (Elendner and Trimborn, 2016) [10].

6. The demand and supply of the stocks are not limited and can be made available by the companies anytime; however, cryptocurrencies are limited in numbers. It is also reported that mining of bitcoin will stop once they reach 21 million in the numbers. Due to this scarcity, asset price becomes so volatile and attracts more customers with additional risk factors as per (Elendner and Trimborn, 2016) [10].

As discussed in above paragraph, such kind of interesting facts in cryptocurrencies or bitcoin add new dimensions in the asset trading based on behavioural finance. New driving force associated with cryptocurrency market goes hand in hand with its' complex nature since it causes speculations in the market.

That's why we have chosen to introduce new parameters related to market factors such as volatility for an asset to study bitcoin price movement using Statistical Modelling approach rather than Artificial Intelligence.

## 2.2 Definition of Bubbles and Crashes

Many researchers and economists have been studying speculations and bubbles happening in stock market for many years. According to Kindelberger (1978) [3], a bubble can be termed as the price movement of an asset in upward direction over a span of time and then sudden plunge occurs. Another definition, by Eatwell and his colleagues (1987) [9]; a bubble is a sign of sharp increment in the price of an asset in the market with a process of consistent movement of price increasing the expectation of more and more participants and then causing sudden depression.

According to Eatwell, Milgate, and Newman (1987) [9], there are participants in the market who does not care about earning nature of an asset but they only care about their own profits, such individuals cause speculations in the market, resulting in bubbles and crashes.

As Garber (2000) [15] stated that the reason for bubble to exist in an asset could be the price deviation from the real worth of an asset. There can be many driving forces such as more flow of cash, discounts causing bubble to occur for that asset in the market. This is also very popular notion among economists that market fundamental factors can provide valuable information to study an asset but only for those assets which follows linear growth over a period of time. Also, such factors have capability to explain random turbulence in stock price for very short duration. Bubbles are results of deviation in the price from the market price over the time period. These bubbles follow exponential growth and can not be explained just by random shocks behaviour in the stocks.

Rosser (2000) also stated that no one is fully aware about such fundamental factors causing bubbles. Such bubbles created difficulty for economists to study and actual behaviour cannot be predicted if speculation is dominating in the market [32].

Siegel (2003) had also suggested new definition for the bubbles, a time period 't' in which price of an asset falls or rise and this price does not follow any consistent pattern or return from the asset is more than 2 standard deviations from the expected return of that asset considering risk involved and return capacity of that asset at time 't'. In a nutshell, it can be said that individuals should have patience to wait before they can predict bubbles [34].

According to this definition, once this time 't' is reached by which price return is 2 standard deviation away from expected return, one can expect bubble. Siegel himself tested this definition to the real price index for equity market data for 25 years. However, this definition had failed to provide evidence to predict a bitcoin bubble since its price is highly volatile and it does not follow any specific pattern. Bitcoin is in existence for less than 12 years by now. The bitcoin price was quite neutral before 2014 and then price surges like never. Hence this definition will not provide us enough evidence since historical bitcoin data will not give true picture of return on bitcoin over the years [34].

Keynes (1930) also put forward his thoughts in term of the capacity of an asset to be converted into cash and he suggested to measure any asset liquidity by the risk associated with that asset and the ability of market to soak high volumes of that asset for sale without much deviation in the price even in the short duration [23].

Further, Yamey (1989) highlighted the key relationship between the volume of an asset and the soaking capacity of the market. If participants keep on trading high volumes such as buying and selling for a longer period, this will result in price movement of that asset. There is term defined for individuals who always engaged in relative high number of transactions, called "Floor Traders". Hence these traders keep on doing more transactions, attracting more customers and speculators to invest heavily. This had happened in case of bitcoin price movement post 2014. Now a sentiment called 'get-rich-quick-mania' becomes so prevalent in the market that a crash followed just after the bubble [35].

This kind of phenomenon was explained by Siegel (2003). These highly investing individuals focus more on selling the asset to gain profit. The price keeps surging as more and more customers are buying it and then another sentiment called panic-mania got triggered among participants. Again, same kind of pattern can be expected from these individuals as all of them started selling together that results in crash [34].

We can divide factors into two types such as endogenous factors which have been driving market and being studied so far. Another type of factors such as exogenous which were studied by Johansen and Sornette (2010) when they built JLS factor model, but they did not introduce exogenous factors to study bitcoin bubble and crash. We will be studying these factors directly with New JLS factor model implementation and will produce the results in later sections.

## 2.3 Bubble Life Cycle

Rodrigue et al. (2009) has defined phases for bubble speculation and there are four stages. First phase is called stealth one in which very few participants who have had more knowledge and information about market fundamental factors driving prices upward. Then, more players got involved and many transactions will happen with great influx of fund. This phase attracts more and more investors and it is called awareness phase [7].

After all this happened, investors and other media people try to create more hype among other participants, resulting in volatility and speculations in the market. Now, emotions and sentiments pitched in among participants by negating all market principles. This phase is called mania phase. Such a positive sentiment toward an asset will trap investors and then sudden decline results in crash of the market. This is called blow off phase. Although an asset price returns to normal what was before the bubble, a lot of turmoil, panic and burst have already happened among investors. Such kind of pattern will affect confidence of participants in a certain asset. These phases have been shown in below Figure 2.1.

If we look at the price movement of the bitcoin bubble phase and phases defined by Rodrigue et al (2009) in below Figures 2.1 and 2.2, there is quite similarity between both the Figures 2.1 and 2.2. In 2017, the bitcoin price arose by 900% and suddenly it was of much attention from the investment perspective in year 2017 among investors as reported by The Guardian, 2018 [1]. Also, the price of bitcoin dropped by 64% (from \$19,163 to \$6,899) from December 17, 2017 to February 6, 2018. This period was called crash phase of 2017. It is quite evident that the time frame between 2014 and 2018 was of high volatility as much as \$3876 on closing price of bitcoin on daily basis [7].

Keynes (1930) had already suggested that an asset associated with high risk might bring a speculative financial bubble in the market [23].

Figure 2.1: Phases of Bubble

Source: Rodrigue et al. (2009).

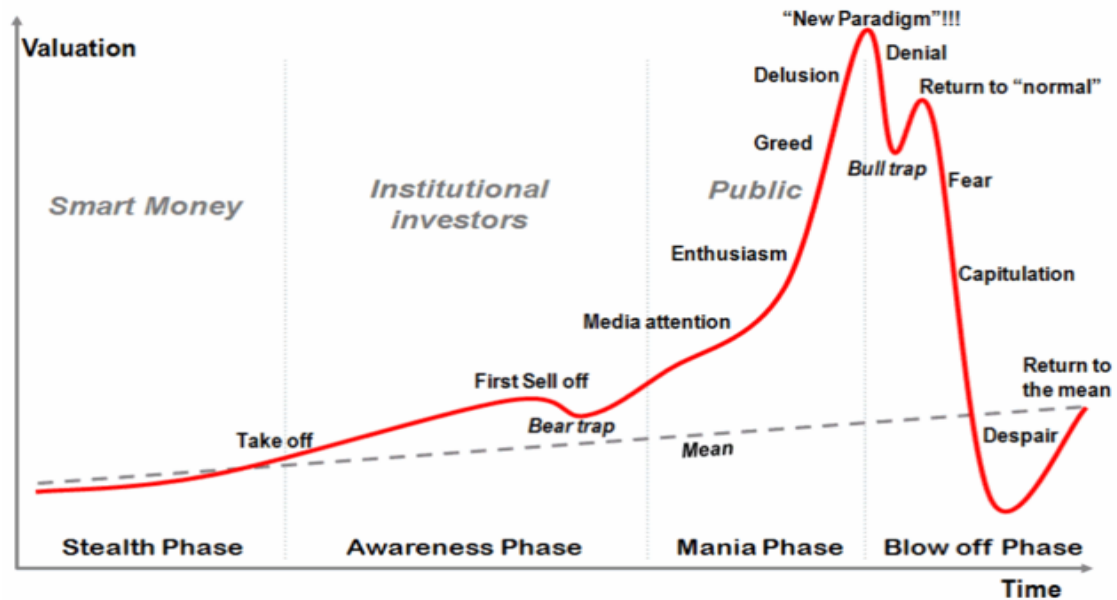
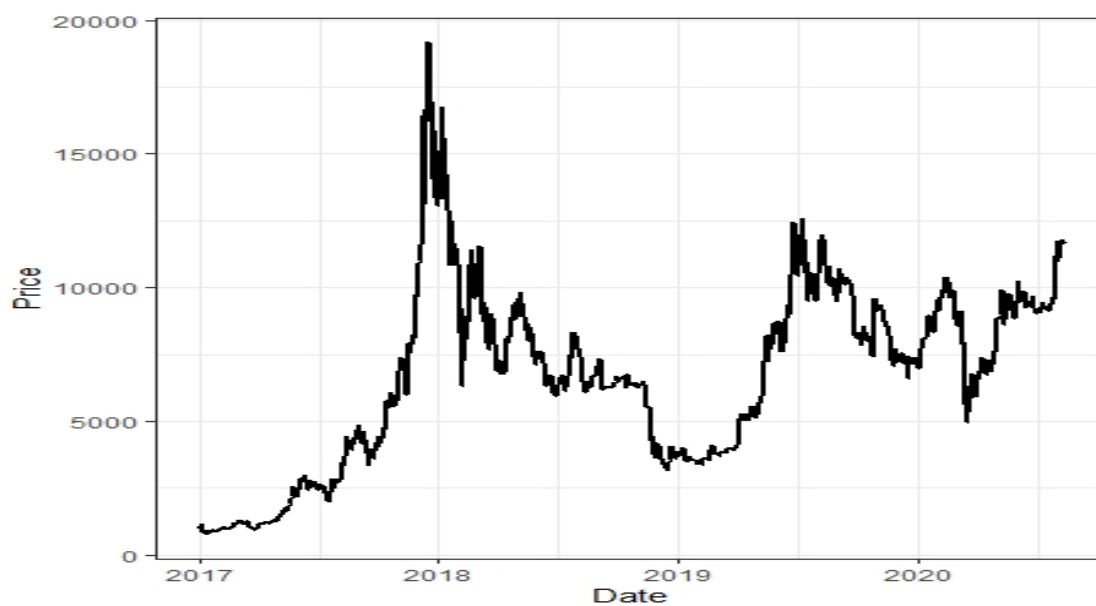


Figure 2.2: Bitcoin Price Plot 2017-2020



## 2.4 Bitcoin

The concept of bitcoin and its existence can be attributed to the technologies forming digital money ecosystem. The participants in the bitcoin network use it as a currency to perform trade and communications is done via Internet. Bitcoin network takes advantage of protocol which can run over desktop or laptop by open source softwares .

Bitcoin is considered fast, secured, digital and new currency to perform trades since participants do not need to be physically present in order to perform transactions. Also, transactions can be performed without border restrictions [5].

Bitcoins are entirely digital. We don't need any physical money to purchase or sell bitcoin. The participants should have key combination to perform trade and authenticate themselves in the network. These keys can be used to validate any transaction and will establish owner's authenticity in the network. These keys can be stored in a virtual wallet on user's phone or computer. Keys are the only requirement to trade in bitcoin.

### 2.4.1 Technology Aspects of Bitcoin

Bitcoin is a peer-to-peer distributed system. Bitcoins are created by a process called mining. In mining process, nodes will be involved to find solutions to a mathematical problem while performing transactions. Any participants can work as a miner by using his computer's processing power in order to validate and record transactions. These transactions are validated in every 10 minutes for past transactions on an average by a miner. He is awarded new bitcoin for such activity. Usually, the protocol used in bitcoin will half the rate at which new bitcoins are generated in each 4 years, restricting the total sum of bitcoin to be generated below fixed 21 million coins by the year 2140. The protocol used is also termed as bitcoin, a peer to peer network and computation in a distributed and innovative way. Bitcoin network can be divided in four parts [5].

- A decentralized peer-to-peer network.
- The block chain .
- Rules for independent transaction validation and printing currency.
- A process to reach global decentralized agreement on the authentic block-chain (Proof-of-Work algorithm).

A series of digital signatures are used for electronic coin transaction. Each bitcoin owner will have to transfer the coin to the next customer by signing digitally a hash of the former transaction and public

key of the next becoming owner. This is added to the end of the coin. Now a payee can validate these signatures to validate series of ownership. There is no way to verify if the owner did not double-spend the coin and payee find it difficult to verify in such scenario. Hence Block chain, a decentralized public ledger to record all verified transactions, is proposed [29].

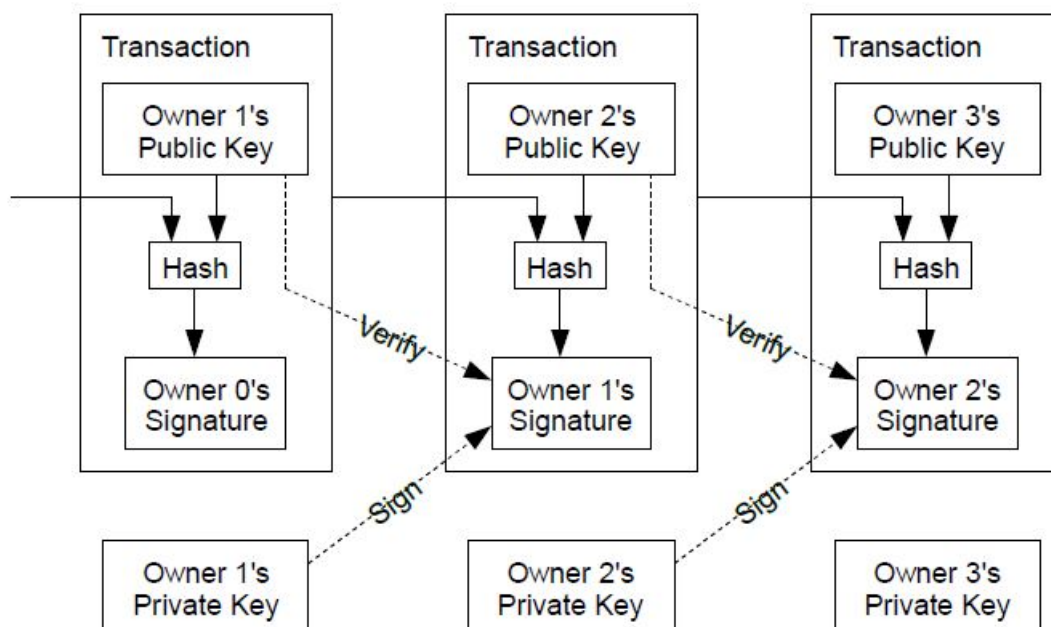


Figure 2.3: Diagram for digital signature

Source: Bitcoin: A Peer-to-Peer Electronic Cash System, by Satoshi Nakamoto

In order to understand how the technology set-up is done in case of bitcoin. A timeserver is used to generate the hash of a block, which has information from the previous block and any type of transaction between two time periods. This timeserver will then broadcast the newly generated hash over the network. By this way, transaction chains are formed with latest block which will strengthen the previous block. After becoming the part of the chain, a specific block can not be modified without affecting other blocks in the chain. By this way, this chain design ensures that data cannot be modified easily, and records integrity is maintained.

In order to implement a distributed timestamp server on a peer-to-peer basis, a proof-of-work system similar to Adam Back's Hashcash [6] is utilized. In this proof-of-work, hashed value is scanned since it starts with several zero bits. The amount of work required to scan this hashed value is quite exponential to the number of bits and can be validated using a single hash. In case of time-stamp network, this proof-of-work can be implemented by increasing a nonce count in the block unless a value is found which provides required zero bits for the hash. Also, since a lot of computation power is involved and CPU effort has already been spent on Proof-of-Work, now the modification of the block

is not possible without repeating the entire work. Since all following blocks are part of the chain, the work will involve redoing the work for all the blocks which is cumbersome effort.

The members in the network will fight during Proof-of-Work mechanism in order to simplify mathematics problems which are difficult to solve. Also, all such bitcoin transactions are verified within 10 minutes of interval and collected into a single block. Since nodes will be competing against each other in PoW to simplify the problem for a transaction, the first one who will solve it will publish the solution to the entire network. Once most of nodes have verified PoW, then bitcoins are issued to that node as an award, called subsidy for the node. This process is termed as mining [29].

The steps to run the network are given below.

1. Spreading all new transactions to all nodes.
2. A block will be formed by each node in process of gathering new transactions.
3. A complex proof-of-work will be figured out by each node for its block
4. After a proof-of-work is found, each node will broadcast the block to all nodes.
5. If all these transactions are found valid and are not double spent in the block, the block will be accepted by Nodes.
6. By utilizing the hash of the previously acquired block, the block will be accepted by the Node in process of generating the next block in the chain.

The process of rewarding the node for validation purpose considered a profitable usage of processing power. As we all know that Bitcoin network processing power has reached to the processing power of 2 trillion laptops in January 2017, this power is two million times larger than the power of world's largest supercomputer. By inventing processing power in such a way, Bitcoin has been termed as the largest single-purpose network in the globe.

Bitcoin transaction happens through digital keys, bitcoin addresses, and digital signatures. These keys are generated and stored by users in a file, or in a database, called a wallet. The protocol used by bitcoin transaction does not define these keys at all and they are independent. These keys are created by the software running inside wallet or database without connecting to internet [5].

The bitcoin transaction can be done using an authentic digital signature created using another secret key. This signature is termed as witness to perform transaction since this signature is making sure to establish the real ownership of the money to be spent.

Now, let's talk about these keys. There are 2 type of keys: private key and a public key. Usually keys are stored inside the file and taken care by software running inside wallet. The cryptography

is employed to generate these key pairs for access protocol of bitcoin. Transactions are signed by private key while money transfer is done using public key. In case of such cryptography algorithm, private keys create signs on encrypted message while authenticity can be checked using public key. All this is possible since there is a mathematical relationship between both the keys [5].

The entire mechanism is shown in below Figure 2.3.

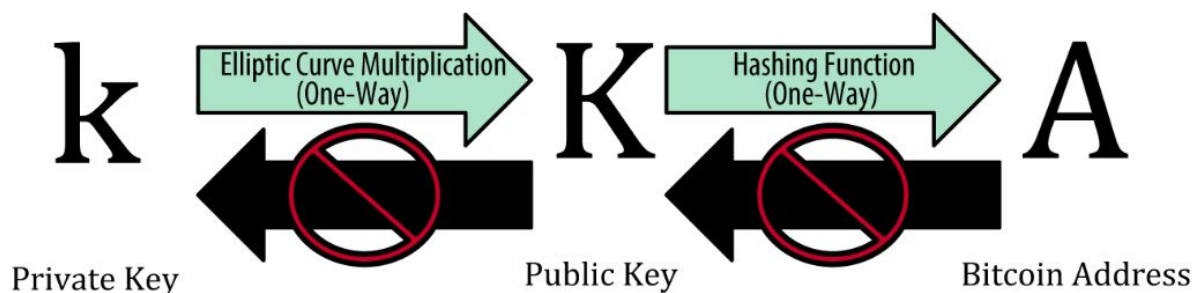


Figure 2.4: Process to Create Public Key and Bitcoin Address

Source: Mastering Bitcoin, by Andreas M. Antonopoulos

The entire transaction can be understood by using below example. If I am the current owner of the bitcoin and I want to spend it. I will present my public key along with signature for transaction to happen in order to spend my bitcoin. These signatures will differ each time but I have used same private key to generate them. Other participants in this network can validate and allow this transaction to happen by the confirmation of my ownership with bitcoin. In this process, recipient will know that I was owner of my bitcoin before the transfer was initiated.

In a nutshell, authenticity, integrity and chronology of these kind of transactions in the bitcoin are ensured by cryptographical algorithm and blockchain technology. In initial days, there were third-party institutions who were making these transactions authentic, but Bitcoin technology made this process easy without the need of any third-party institution but with the usage of wallet and cryptography [5].

## 2.4.2 Bitcoin as a Digital Cash

Since payments and ledger for records are done via computer and networks by banks and other firms. They don't contribute anything to create new form of money. Bitcoin was introduced as alternative digitalised solution to solve problem of money. It has been operating without any problems for the past 11 years and if it will continue to function like this for another 100 years, this will be proved to be a new way to deal in money by replacing paper money. It will provide supreme authority over money without unexpected inflation. Bitcoin has many properties such as highly scalable in space, scale and

time as a digital currency.

In order to understand digital cash, we can divide modes of payment into two distinct sections [4].

1. Cash payments: They are done between two individuals in person. The action is quick and final and does not need any kind of relationship between individuals. No delay is expected from either side and there is no need to involve third-party. The problem is to have presence of both the parties at the same place at the same time.
2. Inter-mediated payments: Such payments do involve third-party or can be in form of cheques, credit cards, debit cards, wire transfers and other ways. It does not need any physical presence of both the individuals at the same time in the same place. However, the trust is the main asset here since money transaction can only performed if individuals allow third-party to perform transaction on their behalf. These third-party agencies charge for time and cost of the transaction to execute the final payment.

Bitcoin, a digital cash, is the first type of digital money solution that allows digital payments without any trust relation or any third-party involvement. As far as we believe, this must be the motivation of Satoshi Nakamoto to create a peer to peer electronic form of cash without any trust relation between parties before the transaction and without the risk of any third-party involvement. Also, the risk of modification by third-party is mitigated. Bitcoin brings the same feature of physical cash in form of digital cash. It is the world's first decentralized digital party to party money transfer mechanism and will proved to be a revolution in digital world soon [4].

Initially, a lot of criticisms was made against bitcoin due to many factors, but investors and economists started understanding technology and bitcoin over time. There are many benefits associated with Bitcoins [26].

1. Bitcoin transactions are fully anonymous and private. Bitcoin transactions cannot be identified by any means but the transaction done by the banks can be tracked.
2. The payment process via bitcoins gives the full freedom. Bitcoin can be transferred to any person in any part of the world with no involvement of third parties and with no bank holiday and no boundaries involved as such.
3. The payment through bitcoin has very low transaction fees. It all depends on the urgency of the individual.
4. Bitcoin transactions are fully secured, can be reversed, and do not contain any personal information of the party.

5. Bitcoin transactions are very quick in comparison to banking transactions. A bitcoin transaction can be performed within 10 minutes and its transaction speed can be compared with email speed.
6. Bad players can steal our payment information from the merchants. Most online transactions today are performed via credit cards, debit cards, ask us to provide all confidential details into a website. Bitcoin transactions don't need any kind of confidential information. It takes advantage of two keys: a public key, and a private key.
7. Bitcoin does not cause inflation.

Though Bitcoin has many advantages, but it draws criticisms due to many factors as well. Many people are still unknown to bitcoin traded as a digital cash. Bitcoin price are highly volatile and can move up and down at very quick pace, causing more panic among participants. Due to high volatility in the market, bad market players try to speculate prices in order to create bubbles. Economists themselves think bitcoin very risky. There are still many enhancements to be done in bitcoin software.

Bitcoin is used in mal practices such as money laundering and in black markets. In case of money laundering, third parties will try to collect money from one party and then send it to another party via bitcoin. Due to anonymity among users, it reinforces prevalence of illicit transactions in speedy and easy way.

According to Foley et al. [13], almost one fourth of the users and half of the bitcoin transactions are involving illicit actions. One such example is silk road which was used as the black market version of ebay between year 2011 and 2013. This black market used 'Tor' network and bitcoin as a payment mode to keep illicit business on the website secret. Since location and real identity of the users cannot be established. As per the reports from FBI and U.S. Attorney's office, almost 173,991 bitcoins were used in Silk Road case, valuing around \$33.6 million at that time [12].

### **2.4.3 Bitcoin as an Investment Asset**

Bitcoins are virtual assets that have gain popularity among customers in last 10 years due to its many features though the price movement behaviour of bitcoin is still unknown to many researchers. As we know that bitcoin has reached unprecedented growth when it created a bubble in 2017 and attracted major proportion of investors followed by a crash. We also know that the overall market capitalization related to cryptocurrency has touched almost \$630 billion. Bitcoin can be traded on different types of cryptocurrency exchanges and price varies a lot due to its demand among participants and then supply of bitcoin to those investors. Over the years, the massive growth of bitcoin and the good return rate on the traded price has attracted many managers from portfolio domain to study it and add bitcoin

as a part of portfolio in order to gain more returns with less risk. We have done our extensive research from different sources and books, hence we are presenting here few facts related to bitcoin as an investment asset [14].

- As we know that bitcoin is extremely volatile due to less regulations on its trading. It is not under any such control the way other stocks are regulated in the stock market. It is also interesting to know that very few know the price dynamics of the bitcoin, causing it more risky than other type of currencies and stocks. It is also very well known that market participants are less likely to invest in the asset which has high volatility with high risk involved. They can be also called risk-averse customers.
- It is a notion among customers that bitcoin price results in bubble and then sudden plunge, making it more attractive among risk-taking investors; however, such kind of bubble are very risky and does not stay for longer period to time. Then, a crash happens in bitcoin price. This brings bitcoin popularity in question for longer period of time and it is less likely that it can be considered a best bet for investment.
- As we have discussed so far about bitcoin which add diversity in investors' portfolio, it brings benefits to participants portfolio in short duration or in long duration. Bitcoin shows low level of correlation coefficient with other types of stocks and assets such as gold, oil and S&P500. Bitcoin brings good returns on investment with risk being adjusted to the portfolio (Platanakis and Urquhart, 2019) [31]. According to Koutmos (2019) [25], bitcoin price movement is quite affected by fundamental economic factors such as interest rate, volatility of the asset and other sentiments. Also, many researchers have been comparing return rates by investing in gold with return rates from bitcoin. According to Dyhrberg (2016) [8] study, Bitcoin can be traded from the perspective of hedging fund as the gold shares the same abilities. However, another study done by Klein et al. (2018) [24] raises questions on hedging features of bitcoin in longer term, termed bitcoin as not very safe asset.
- The bitcoin price movement is not independent of historical price. This is also called as Weak market form efficiency. Bitcoin current price shows the trend which can be attributed to the price in the past. According to Phillip et al. (2018) [30], bitcoin price shows long memory to be associated with its historical data and it takes time before such effect feeble. There is clear relation between strong market inefficiency and bitcoin price. But the strong belief among researchers suggests that such market related inefficiencies will vanish in the future.

Overall, we deduced that bitcoin can be traded as an investment asset, but this asset is dependent on various market factors. Participants will repose their confidence by investing in bitcoin

if bitcoin market is more legalised. There should be more bitcoin related policies and laws with less malpractices to instil the confidence of investors in the bitcoin. When more players will trust this as an investment asset, the market capitalization will keep on improving automatically. Also, cryptocurrencies should be regulated by the tax in order to make it a safe asset and it will bring attention of customers from security point of view as well rather than bitcoin bubble influence on the investors' mind. There is still a lot of insecurity and complexity causing investors to stay away from cryptocurrency. According to Mainelli and Smith (2015) [28], block chains may be used to challenge the secrecy of the transactions, resulting more and more doubts about bitcoin future. Hence this problem should be addressed in order to increase Bitcoin transaction in future.

## 2.5 Statistical Models for Bubble Prediction

### 2.5.1 ARMA Model

Earlier, Auto Regressive Moving Average (ARMA) model was used to predict bitcoin price movement. In order to do prediction, the historical price for bitcoin was selected between year July 18, 2010 and August 25, 2013. ARMA model is based on a theorem which states that any kind of discrete random price can be categorised into 2 separate processes given as, autoregressive and moving average. Firstly, a univariate ARMA model was derived but it didn't generate better results and then, multivariate model was used to incorporate more parameters for better prediction. This model was able to follow prediction for few days but failed to predict any bubble and crash. Also, this model kept exogenous market factors to be constant which is not the intention of our research paper [27].

### 2.5.2 JLS Factor Model(LPPL)

LPPL(log periodic power law) model had been used widely by many economists in order to predict bubble and crash since many years. Since bitcoin is highly volatile asset and its price movement follows faster-than exponential growth. Many researchers have defined specific time window in order to predict bubble, crash and future price movement. Recent work was done to predict bubble in the year 2017 and then price prediction post September 2019. This model describes the bubble behaviour the best way and prediction is also matching with actual price with a given confidence interval. The time to predict the critical time was nearly in the range when the actual bubble happened in the year 2017. However, this model is based on second order JLS model which does not involve any exogenous market factors such as historical volatility of an asset, interest rate, deposit rate and many more when making prediction. We will be looking at few of these factors in our research area. Also, equation for Old JLS model (LPPL) can be referred from later sections of this thesis [20].

### 2.5.3 New JLS Factor Model

As we have discussed in last section, old JLS model does not talk about historical volatility of an asset when making prediction. This is also very well-known fact that bitcoin bubble can be not studied by using balanced models. Zongyi Hu and Chao Li have studied bubbles for Chinese stocks by extending Old JLS factor model by introducing new factors such as interest rate and historical volatility of the asset over the time period. They built this model based on Chinese market specific factors as Chinese stock price movement was quite volatile and it can be attributed to other market factors such as interest rate, deposit reserve rate and few more. They applied this model to predict bubble for Chinese stock between July 2005 and October 2008, November 2008 and August 2008, and March 2014 and July 2015. After comparison of the results provided by both the models, it was concluded that New JLS factor model was able to predict critical time closer to actual time of bubble for Chinese Stock. I will be using same model to predict Bitcoin bubble and crash [20].

## 2.6 Optimisation Algorithm (CMA-ES)

In case of Deep learning, we efficiently calculate the gradient of an objective function over each model parameter by using back propagation algorithm. This methodology makes Neural Networks highly efficient since we found an optimized solution which is good enough to solve complex tasks. In case of Reinforcement Learning, gradient strategy does not fulfil our purpose since gradient of awarding signal should not be passed to the future agent for an ongoing action now. It is also observed to get stuck in a local optimum for Reinforcement Learning Tasks [18].

In order to get rid of such difficulty, there was a paper published by OpenAI [33] which talks about Evolution Strategies as a scalable approach to Reinforcement Learning in which evolution strategies were used to offer many advantages. These strategies do not use gradient calculation for evaluation purpose and the computation for an ES algorithm can be distributed to thousands of machines for computation in parallel manner. By running the algorithm from beginning many times, it can be shown that policies discovered by Evolution strategy (ES) become wider in comparison to policies found by Reinforcement Learning algorithms. Evolution strategy provides a user a set of eligible solutions to evaluate a problem. The evaluation is done by using an objective function that takes a given solution and results in a single fitted value. Based on the fitted results of the current solutions, the algorithm will be evaluated again to produce the next set of candidate solutions which are believed to produce even better results than the current set of candidates. The iterative process will terminate once the user got the satisfactory results.

In case of Simple Evolution Strategy, we took sample of solutions from Normal Distribution with

mean  $\mu$  and a constant standard deviation  $\sigma$ . The mean is set to the origin and then set to the best solution in the population and then we do sampling in next set of solutions around the mean. This strategy is only successful for simpler problems and usually got stuck the local optimum for more complicated problems.

In case of Simple Genetic Algorithm strategy, we focus only on 10% of the best performing solutions in the current samples and throw away the rest of the population. In the next set, 2 solutions were selected from previous generation randomly and they were combined to produce a new solution. This strategy will keep track of a wider set of solutions to reproduce the next set of solutions.

Both Simple ES and Simple GA have standard deviation parameter fixed. What if we need to explore more and need to increase the space for our research? The right answer is: Covariance-Matrix Adaptation Evolution Strategy (CMA-ES). This strategy focuses on fine tuning the solution even after reaching a good optimum. This technique will not only modify mean  $\mu$  and sigma  $\sigma$  but also generate covariance matrix for the population set. CMA-ES algorithm takes the results of each generation, and make sure to increase or decrease the standard deviation for our searching space for the next generation. It will calculate the entire covariance matrix with the mean  $\mu$  and the sigma  $\sigma$  parameters. At each generation, CMA-ES calculates the parameters of a multi-variate distribution which is normal distribution from sampled solutions. This algorithm is very popular optimisation algorithm without gradient approach and performance is always good when considering less than a thousand parameters [18].

### 2.6.1 Algorithm Description

Before we start dive deeper into CMA-ES algorithm, let us talk about covariance matrix of our entire sampled population of Size 'N'. Firstly, we will calculate means of  $x_i$  and  $y_i$  in the population as given below:

$$\mu_x = \frac{1}{N} \sum_{i=1}^N x_i \quad (2.1)$$

$$\mu_y = \frac{1}{N} \sum_{i=1}^N y_i \quad (2.2)$$

Other terms for covariance matrix will look like as given below.

$$\sigma_x^2 = \frac{1}{N} \sum_{i=1}^N (x_i - \mu_x)^2 \quad (2.3)$$

$$\sigma_y^2 = \frac{1}{N} \sum_{i=1}^N (y_i - \mu_y)^2 \quad (2.4)$$

$$\sigma_{xy} = \frac{1}{N} \sum_{i=1}^N (x_i - \mu_x)(y_i - \mu_y) \quad (2.5)$$

CMA-ES will help to modify this original formula to use it for an optimisation problem. Firstly, CMA-ES will pick the best  $N_{best}$  solutions in the current set of solutions which can be the best 25% of solutions. Once solutions will be sorted, the mean  $\mu^{(g+1)}$  of the next generation ( $g+1$ ) can be calculated by averaging the only best 25% of the solutions in current population ( $g$ ) as given below:

$$\mu_x^{(g+1)} = \frac{1}{N_{best}} \sum_{i=1}^{N_{best}} x_i \quad (2.6)$$

$$\mu_y^{(g+1)} = \frac{1}{N_{best}} \sum_{i=1}^{N_{best}} y_i \quad (2.7)$$

Now, we can use only the best 25% of the solutions to calculate the covariance matrix  $C^{(g+1)}$  of the next set of solutions, but it will use the current set of solution's  $\mu^{(g)}$ , rather than the updated  $\mu^{(g+1)}$  parameters that we had just calculated [18].

$$\sigma_x^{2,(g+1)} = \frac{1}{N_{best}} \sum_{i=1}^{N_{best}} \left( x_i - \mu_x^{(g)} \right)^2 \quad (2.8)$$

$$\sigma_y^{2,(g+1)} = \frac{1}{N_{best}} \sum_{i=1}^{N_{best}} \left( y_i - \mu_y^{(g)} \right)^2 \quad (2.9)$$

$$\sigma_{xy}^{(g+1)} = \frac{1}{N_{best}} \sum_{i=1}^{N_{best}} \left( x_i - \mu_x^{(g)} \right) \left( y_i - \mu_y^{(g)} \right) \quad (2.10)$$

Lets start with full implementation of CMA-ES [19]. In the initial step of the algorithm, we will need to set the initial point  $m_0 \in \mathbb{R}^n$ , the initial step size  $\sigma_0 > 0$  and the number of individuals at each iteration  $\lambda$ . In the CMA-ES strategy, a sample of a multivariate normal distribution will define a population of points. The basic equation for sampling the search points, for generation number  $g = 0, 1, 2, \dots$  is represented by.

$$x_k^{g+1} \sim m^g + \sigma^{(g)} N(0, C^{(g)}) \quad (2.11)$$

for  $k = 1, \dots, \lambda$

where

$\sim$  denotes the same distribution on the both sides.

$N(0, C^{(g)})$  is a multivariate normal distribution with zero mean and covariance matrix  $C^{(g)}$ .

$x_k^{(g+1)} \in R^n$ :  $k^{\text{th}}$  individual search point from generation  $g + 1$ .

$m^{(g)} \in R^n$ : mean value of the search distribution at generation  $g$ .

$\sigma^{(g)} \in R^n$ : standard deviation while generation  $g$ .

$C^{(g)} \in R^n$ : covariance matrix at generation  $g$ .

$\lambda \geq 2$ : population size, sample size.

The new mean  $m^{(g+1)}$  of the distribution is calculated as the weighted average from the given sample  $x_1^{(g+1)}, \dots, x_\lambda^{(g+1)}$

$$m^{(g+1)} = \sum_{i=1}^{\mu} w_i x_i^{(g+1)} \quad (2.12)$$

$$\sum_{i=1}^{\mu} w_i = 1$$

, where  $w_1 \geq w_2 \geq \dots \geq w_\mu > 0$

$\mu \geq \lambda$ : the number of selected points.

$w_{i=1 \dots \mu} \in R$  : positive weight coefficients for recombination.

Equation 2.12 calculates the mean value of  $\mu$  selected points. Then, the selected points are sorted so that higher weights are assigned to that sample which optimizes objective function  $f$ , i.e.  $f(x'_{(g+1,1)}) < f(x'_{(g+1,2)}) < \dots < f(x'_{(g+1,\mu)})$ . By this way, it can be assumed that the selected points are much closer to the optimal point.

The final equation for updated  $m$  is given as below:

$$m^{(g+1)} = m^{(g)} + c_m \sum_{i=1}^{\mu} w_i (x_{i:\lambda}^{(g+1)} - m^{(g)}) \quad (2.13)$$

where

$c_m \leq 1$  is a learning rate, usually set to 1.

Now we shall derive the update of the covariance matrix,  $C$ . We can estimate the covariance matrix from a single population of one set of solutions. Such kind of estimation is not very reliable for smaller set of samples and another way has to be used such as rank- $\mu$ -update. Since it can be assumed that

the population has sufficient information to calculate a covariance matrix. Now we can re-estimate original covariance matrix from the sampled population using below equation.

$$C_{emp}^{(g+1)} = \frac{1}{\lambda-1} \sum_{i=1}^{\lambda} \left( x_i^{(g+1)} - \frac{1}{\lambda} \sum_{j=1}^{\lambda} x_j^{(g+1)} \right) \left( x_i^{(g+1)} - \frac{1}{\lambda} \sum_{j=1}^{\lambda} x_j^{(g+1)} \right)^T \quad (2.14)$$

We can also use different approach to estimate  $C^{(g)}$  as given below.

$$C_{\lambda}^{(g+1)} = \frac{1}{\lambda} \sum_{i=1}^{\lambda} \left( x_i^{(g+1)} - m^{(g)} \right) \left( x_i^{(g+1)} - m^{(g)} \right)^T \quad (2.15)$$

$C_{\lambda}^{(g+1)}$  is called unbiased estimator of  $C^{(g)}$  [19]. The difference between these two equation is called reference mean value.  $C_{emp}^{(g+1)}$  is used for the mean of the actual samples.  $C_{\lambda}^{(g+1)}$  is used for true mean value  $m^{(g)}$  of the sample. In a nutshell, it can be said that  $C_{emp}^{(g+1)}$  calculates the variance within the samples points while  $C_{\lambda}^{(g+1)}$  calculates variances of sampled steps. Also, covariance matrix can be re-calculated by introducing weighted selection mechanism as given below.

$$C_{\mu}^{(g+1)} = \sum_{i=1}^{\mu} \left( x_{i:\lambda}^{(g+1)} - m^{(g)} \right) \left( x_{i:\lambda}^{(g+1)} - m^{(g)} \right)^T \quad (2.16)$$

$C_{\mu}^{(g+1)}$  can be referred as estimator for selected steps. It is likely to reproduce the successful steps. It is also noted that covariance matrix adaptation in increases or decreases the scale in a single direction for every step. It does it by removing the old information. Usually three methods are used in step control:  $1/5^{\text{th}}$  success rule,  $\sigma$ -self-adaptation rule and cumulative path length control.

Cumulative step length adaptation (CSA) method can be used to control the step size. When the path of evolution is short, the steps try to cancel each other out as given below in left sub-figure and the step-size should be reduced. When the path of evolution is long, the steps will point to similar directions as given in right sub-figure. Since the steps are similar, the steps can be taken longer into the same directions. Hence, the step-size should be up. If the steps are perpendicular, they are not correlated as given in centre sub-figure. The algorithm will set evolution path  $p_{\sigma} = 0$  and will also define 2 control parameters such as  $c_{\sigma} \approx 4/n$ ,  $d_{\sigma} \approx 1$ . Now, step size can be estimated using below equations [19].

$$\rho_{g+1,\sigma} = (1 - c_\sigma) \rho_{g,\sigma} + \sqrt{\mu_w} \sum_{i=1}^{\mu} w_i x'_{g+1,\mu} \quad (2.17)$$

$$\sigma_{g+1} = \sigma_g \exp \left( \frac{c_\sigma}{d_\sigma} \left( \frac{\| \rho_{g+1,\sigma} \|}{E \|N(0, I)\|} - 1 \right) \right) \quad (2.18)$$

The given algorithm will keep on executing the aforementioned three steps to update three parameters  $m_{g+1}$ ,  $C_{g+1}$  and  $\sigma_{g+1}$  sequentially, until we reached to an optimal solution. This can be implemented using 'CMA-ES' package provided by R.

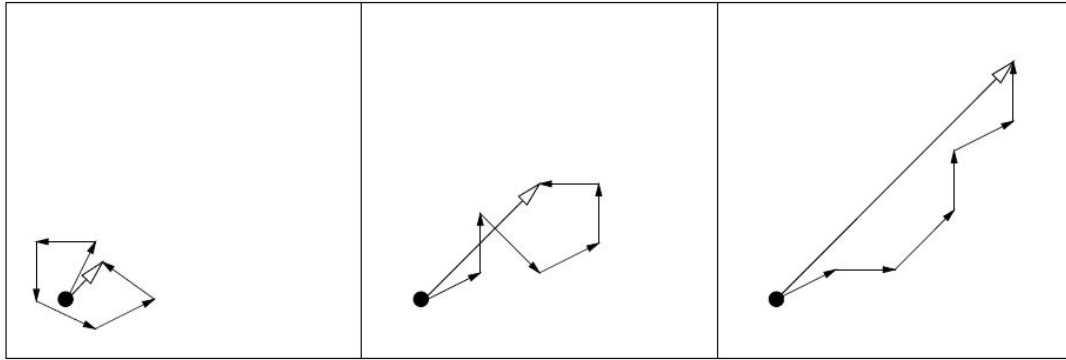


Figure 2.5: Different Evolution Path

## Chapter 3

# Research Question

As we know that the stock market and bitcoin price movement are not easy to explore, studying bubbles in any stock market is also not independent of other stocks. Thus, using a balance model is not a right approach to study any kind of bubble. Since there are limitations in classical financial theorem, there was a lot of research involved to understand the price movement of bubbles in stock markets. According to Scholars, Sornette and Johansen, bubbles are not defined by an exponential increase in prices but rather by they follow a faster-than exponential growth in prices. They proposed a second order model called LPPL(JLS) model [21].

Also, Johansen and Sornette [21] argued that a large crash is the result of the local self-reinforcing imitation between customers and their herding behaviour. Since customers want to maximise their chances to gain profit and get influenced by others, this behaviour eventually leads to a bubble. Generally, it is believed that participants try to copy their other friends in the market up to a critical point and many of them will sell the order at the same time, resulting in the market crash. Johansen and Sornette also suggested a third-order equation in which demand reduced gradually with limiting factors that leads to a diminished power law of the market price. Then a gradual reduction in log-periodic oscillation will lead us to Anti-bubble. This model was applied on many stock markets such as Japanese Stock and Chinese Stock Market.

Unfortunately, the old JLS model is based on the fact that these bubbles or crashes are the by-product of the interactions among participants in the market due to their imitating behaviour, also called endogenous factors, while exogenous factors (fundamental factors) hardly affect bubble or price movement in the stock market, which is not completely true as per economy theory.

It was also found in one of the studies that there is the impact of government policies and other exogenous factors on the formation of stock bubble such as in the paper which talks about the Chinese stock price movement with other market factors, esp. the effect from changes in policy specific to monetary terms and volatility in international stock markets. Hence it was decided to incorporate the

standard JLS model with more fundamental economic factors in doing study for Chinese stock such as the interest rate, deposit reserve rate and the historical volatility of targeted asset. The results presented by a study was commendable. This new model was named New JLS factor model after successful prediction on Chinese stocks [20].

Hence, we will study the same model with factors relevant for bitcoin price movement to validate if bitcoin bubble and crash can be predicted in the year 2017 and 2020. Since artificial intelligence is quite prevalent to predict stock price movement but our focus is to apply statistical model on bitcoin data and this process is simpler in comparison to other methods. As bitcoin data is independent of the interest rate and deposit reserve rate, we will be incorporating historical volatility of bitcoin in our New JLS factor model to validate if prediction can be made closer to actual bitcoin price.

## Chapter 4

# Model Description

### 4.1 New JLS Factor Model Derivation

As we have already seen that the stock market is a complex system and market can not be studied using a balanced model. There were many researches done to study stock market using non-linear equations to predict the stock price. In order to study stock bubble and crash, there was a research done by Sornette and Johansen who established that bubbles don't follow an exponential increase in price rather they follow faster than exponential growth. In order to follow such pattern, they had given a model called log-periodic power law model (LPPL). This LPPL model was earlier used in statistical physics to study critical points such as magnetism or seismology. Later, this model was called original Johansen-Ledoit-Sornette (JLS) model. We will start with deriving equation for Old JLS model and then derive final equation for our New JLS factor model.

To derive Old JLS model, it was assumed that an asset does not have any dividend and will not be influenced by interest rate, risk aversion and other constraints. The asset follows a martingale process [17]:

$$E_t[p(t')] = p(t), \forall t' > t \quad (4.1)$$

Where  $p(t)$  represents price of an asset at time  $t$ . where  $E_t$  represents the conditional expectation at time taken all information into consideration. By assuming that if there is zero percent chance of crash to occur, a jump process  $j$  can be assigned a value zero and one after the crash at critical time  $t_c$ . Since value of  $t_c$  is not available and the probability per unit of time of crash called hazard rate can

be obtained by below equation.

$$h(t) = \frac{q(t)}{[1 - Q(t)]} \quad (4.2)$$

In which  $Q(t)$  is a cumulative distribution function and  $q(t)$  is probability density function. It can be assumed that asset price decreases by a fixed proportion  $k \in (0, 1)$ . Equation for asset price changes as given below.

$$\begin{aligned} dp &= \mu(t)p(t)dt - kp(t)dj \Rightarrow \\ E[dp] &= \mu(t)p(t)dt - kp(t)[P(dj = 0) \times (dj = 0) + P(dj = 1) \times (dj = 1)] = \\ &= \mu(t)p(t)dt - kp(t)[0 + h(t)dt] = \mu(t)p(t)dt - kp(t)h(t)dt \end{aligned}$$

Where  $\mu(t)p(t)dt - kp(t)h(t)dt = 0$ , resulting in  $\mu(t) = kh(t)$ . Hence asset price equation changes to:

$$\ln \left[ \frac{p(t)}{p(t_0)} \right] = \kappa \int_{t_0}^t h(t') dt' \quad (4.3)$$

This equation suggests that there will be high chances of crash along with increased rate of price change, causing high amount of risk in form of crash for customers. Further JLS (2000) [21] established that the participants are mutually connected to each other and can be represented by integers as  $i = 1, 2, \dots, n$ . Each participant can exhibit 2 kinds of behaviour such as buying or  $s_i = +1$  or selling  $s_i = -1$ . Each participant position in the market can be given by below equation.

$$s_i = \text{sign} \left( K \sum_{j \in N(i)} s_j + \sigma \varepsilon_i \right) \quad (4.4)$$

Where  $K$  represent the imitating factor of participants.  $\sigma$  represents idiosyncratic behaviour of the participants. This equation only talks about the current situation of a participant at one time and it can be affected by new factors. In a nutshell, a critical point  $K_c$  can define current situation of a participant in the market. As  $K$  approaches  $K_c$ , the imitation factor involved with participants will last for long and new equation can be derived if more participants are taking the same position as given below.

$$\chi \approx A(K_c - K)^{-\gamma} \quad (4.5)$$

Where  $A$  is a positive constant and  $\gamma$  is the factor for market sensitivity. Since  $K_c$  is dependent on time 't', it can be assumed that critical point  $t_c$  happens when  $K(t_c)$  approaches  $K_c$ , or  $K_c - K(t) == \text{constant}$  ( $t_c - t$ ). Johansen et al. (2000) stated that this hazard rate for critical time can be linked to the market sensitivity.

$$h(t) \approx B \times (t_c - t)^{-\alpha} \quad (4.6)$$

However this equation does not provide behavioural tendency of participants globally since many participants are in the market and they will be influencing each other. There will be links among participants and equation can be derived after  $n$  iterations among participants considering each participant will have  $2^n$  neighbors. When  $K$  approaches  $K_c$ , the market sensitivity tends toward infinity and it can follow power law as given below.

$$\chi \approx \text{Re}[A_0(K_c - K)^{-\gamma} + A_1(K_c - K)^{-\gamma+i\omega} + \dots] \quad (4.7)$$

$$\approx A'_0(K_c - K)^{-\gamma} + A'_1(K_c - K)^{-\gamma} \cos[\omega \log(K_c - K) + \psi] \quad (4.8)$$

Where  $A_0$ ,  $A_1$ ,  $\omega$ , and  $\psi$  represent real numbers. Above equation can be re-written in form of hazard rate as below.

$$h(t) \approx B_0(t_c - t)^{-\alpha} + B_1(t_c - t)^{-\alpha} \cos[\omega \log(t_c - t) + \psi'] \quad (4.9)$$

Since hazard rate follows time 't', this rate tend to increase as 't' approaches critical time and above equation will change to below form after addition of increasing oscillations. The bitcoin price can be calculated by below equation before the crash.

$$\ln[p(t)] \approx \ln[p(c)] - \frac{\kappa}{\beta} \{B_0(t_c - t)^\beta + B_1(t_c - t)^\beta \cos[\omega \ln(t_c - t) + \phi]\} \quad (4.10)$$

More generic form can be re-written as follows:

$$\ln[p(t)] \approx A + B(t_c - t)^\beta \{1 + C \cos[\omega \ln(t_c - t) + \phi]\} \quad (4.11)$$

Where  $A > 0$ :  $[\ln p(t_c)]$  values at critical time  $t_c$ .  $B < 0$ , represents the increase in  $[\ln p(t)]$  over the period of time before the crash.

$$B = -\frac{\kappa}{\beta} B'$$

C represents the magnitude of the oscillations.

$$C = \frac{\beta^2 C'}{(\omega^2 + \beta^2)}$$

$\beta$  measures the power law and  $\omega$  represents the frequency at which oscillation moves.  $\phi$  is the phase of oscillation. This whole equation is called LPPL model or Old JLS factor model [20].

$$\ln[p(t)] = A + Bx^\beta + Cx^\beta \cos(\omega \ln|t_c - t| + \phi) \quad (4.12)$$

where  $x = (t_c - t)$

or

$$y(t) = A + Bf(t) + Cg(t)$$

where

$$\begin{aligned} y(t) &= \ln[p(t)], \\ A &= \ln[p(t_0)], \\ f(t) &= |t_c - t|^\beta, \\ g(t) &= |t_c - t|^\beta \cos(\omega \ln|t_c - t| + \phi) \end{aligned} \quad (4.13)$$

It was assumed that  $r(t) = 0$  and  $v(t) = 0$  which do not hold true in real world. New JLS factor model can be defined as given below [20].

$$y(t) = A + Bf(t) + Cg(t) + \alpha r(t) + \varphi v(t)$$

where

$$\begin{aligned}
 y(t) &= \ln[p(t)], \\
 A &= \ln[p(t_0)], \\
 f(t) &= |t_c - t|^\beta, \\
 g(t) &= |t_c - t|^\beta \cos(\omega \ln |t_c - t| + \phi) \\
 r(t) &= \int_{t_0}^t r(\tau) d\tau \\
 v(t) &= \int_{t_0}^t \sigma(\tau) d\tau \\
 \sigma(t) &= \sqrt{\frac{\sum_{t=1}^t (p_t - \bar{p}_t)^2}{(n)}}
 \end{aligned} \tag{4.14}$$

Also,

$$\begin{aligned}
 \int_{t_0}^t r(\tau) d\tau &\approx \sum_{t_0+1}^t \frac{[r(\tau-1) + r(\tau)]}{2} \\
 \int_{t_0}^t \sigma(\tau) d\tau &\approx \sum_{t_0+1}^t \frac{[\sigma(\tau-1) + \sigma(\tau)]}{2}
 \end{aligned}$$

Also,  $r(\tau)$  is defined as risk-free interest rate and  $\sigma(\tau)$  defines the historical volatility of the asset.  $p_t$  is log price of an asset.  $\bar{p}_t$  is average price of an asset on log scale. Since we don't have  $r(t)$  for bitcoin. Hence our final New-JLS factor model can be reduced to below equation as  $r(t) = 0$ .

$$\boxed{y(t) = A + Bf(t) + Cg(t) + \varphi v(t)} \tag{4.15}$$

## 4.2 New JLS Factor Model Parameters

Since we know that there are unknown parameters in New JLS factor model such as  $A, B, C, \beta, \omega, \phi, t_c$  and  $\varphi$  in which we focus more on critical point  $t_c$ . Also, we need to focus on reducing the number of parameters [11] and get away with dependency between  $\phi$  and  $\omega$ . By utilizing this strategy, equation can be re-written as given below:

$$\ln E[p(t)] = A + B(t_c - t)^\beta + C(t_c - t)^\beta \cos(\omega \ln(t_c - t)) \cos \phi + C(t_c - t)^\beta \sin(\omega \ln(t_c - t)) \sin \phi + \varphi v(t) \tag{4.16}$$

We can include two new parameters:

$$C1 = C \cos \phi, \quad C2 = C \sin \phi$$

$$\ln E[p(t)] = A + B(t_c - t)^\beta + C_1(t_c - t)^\beta \cos(\omega \ln(t_c - t)) + C_2(t_c - t)^\beta \sin(\omega \ln(t_c - t)) + \phi v(t) \quad (4.17)$$

As seen in above equation, the number of non-linear parameters ( $t_c, \omega, \beta$ ) are three and there are five linear parameters A, B, C1, C2 and  $\phi$ . Our main objective is to estimate these parameters using least-square method on cost function.

$$F(t_c, \beta, \omega, A, B, C_1, C_2, \phi) = \sum_{i=1}^N \left[ \ln p(t) - A - B(t_c - t)^\beta - C_1(t_c - t)^\beta \cos(\omega \ln(t_c - t)) - C_2(t_c - t)^\beta \sin(\omega \ln(t_c - t)) - \phi v(t) \right]^2 \quad (4.18)$$

This non-linear optimization equation can be derived as in equation.

$$\{\hat{t}_c, \hat{\omega}, \hat{\beta}\} = \arg \min_{t_c, \omega, \beta} F_1(t_c, \omega, \beta)$$

$$F_1(t_c, \omega, \beta) = \arg \min_{A, B, C_1, C_2, \phi} F(t_c, \omega, \beta, A, B, C_1, C_2, \phi)$$

There are 2 benefits achieved by reducing the parameters:

- The complex problem becomes easier to solve by mathematical equation.
- We are able to get rid of irregular periodicity in the cost function due to  $\phi$ . We are no longer in the difficulty of finding multiple minima for the cost function.

## Chapter 5

# Methodology

### 5.1 Collection of Data

We have collected the data from Coin Desk website. This website provides data for other currencies such as XRP, Stellar, Litecoin and Monero etc. We shall be using Bitcoin Price Index on daily basis. This daily price index data is specific to average close price. In order to further analysis, we have defined our analysis window from January 1, 2017 to August 10, 2020. There are almost 1225 observations. Since we have introduced historical volatility in our New JLS factor model, we will pre-calculate the historical volatility of bitcoin and will store with data using below equation as provided in Section 4.1.

$$\begin{aligned} v(t) &= \int_{t_0}^t \sigma(\tau) d\tau \\ \sigma(t) &= \sqrt{\frac{\sum_{t=1}^t (p_t - \bar{p}_t)^2}{(n)}} \end{aligned} \quad (5.1)$$

where,

$$\int_{t_0}^t \sigma(\tau) d\tau \approx \sum_{t_0+1}^t \frac{[\sigma(\tau-1) + \sigma(\tau)]}{2}$$

Also,  $\sigma(\tau)$  defines the historical volatility of the asset.  $p_t$  is log price of an asset.  $\bar{p}_t$  is average price of an asset on log scale.

### 5.2 Data Analysis

We have collected the data from Coin Desk website. This website provides data for other currencies such as XRP, Stellar, Litecoin and Monero etc. We shall be using Bitcoin Price Index on daily

Table 5.1: Summary statistics on Bitcoin Price data

Mean	Median	Min	Max	Standard Deviation
6611.5	6866.9	772.7	19167	3427.229

basis. This daily price index data is specific to average close price. In order to further analysis, we have defined our analysis window from January 1, 2017 to August 10, 2020. There are almost 1225 observations.

The bitcoin price is plotted on daily basis from the year 2017 to the year 2020 as given in Figure 5.1. We can refer to table for summary statistics for bitcoin prices over the years in Table 5.1. As we can see that bitcoin price is highly volatile. For instance, the mean price between year 2017 and 2020 is \$6611.5 and the median price is \$6866.9, showing a negative skewed distribution. It is also evident that maximum price reached at \$19167.0 on December 17, 2017 and the minimum price reached so far at \$772.0 on January 12, 2017. Below formula is used to calculate daily return of bitcoin price data.

$$Return = \frac{Price_t - Price_{t-1}}{Price_{t-1}}$$

The return on bitcoin price on daily basis varies between -27.09 % to 23.93%. We can see in below Table 5.2 that the mean value of return is around 0.31% while the median is around 0.28%. It is quite encouraging for investors as return on bitcoin price is quite high compared to other assets and Bitcoin can be considered an investment asset with risk involved as price changes frequently with good return.

Table 5.2: Summary statistics on Bitcoin Return Price

Mean	Median	Min	Max	Standard Deviation
0.31%	0.28%	-27.09%	23.93%	4.57%

Figure 5.1: Price Plot for Bitcoin

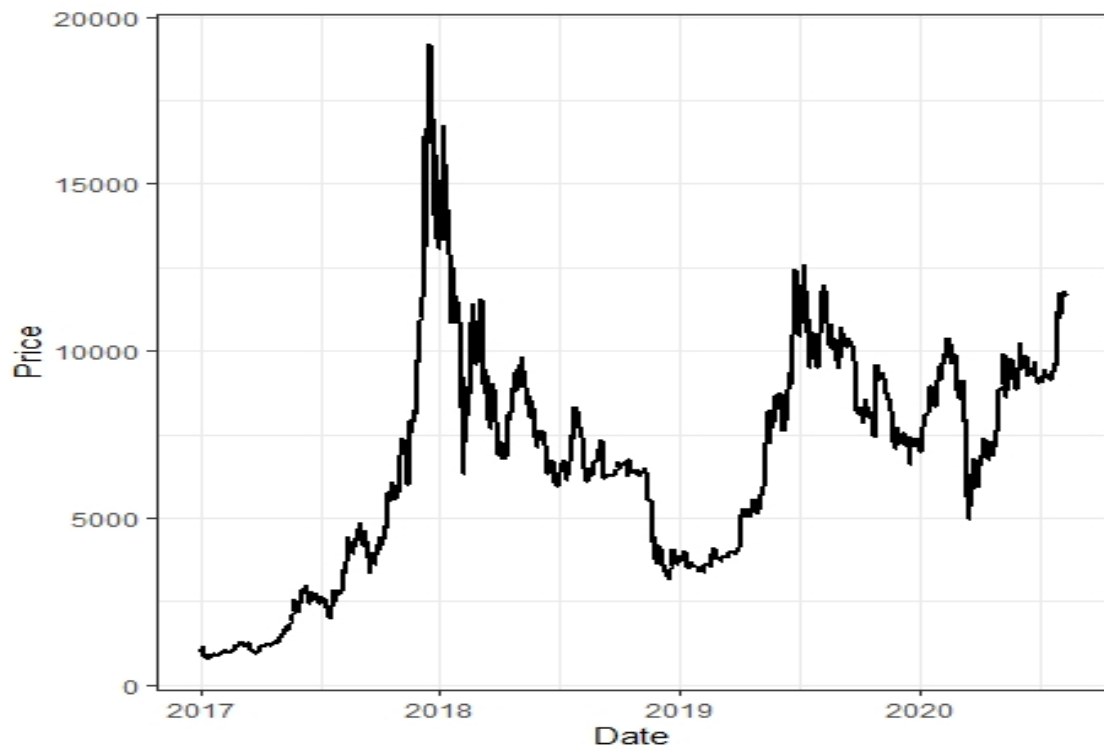
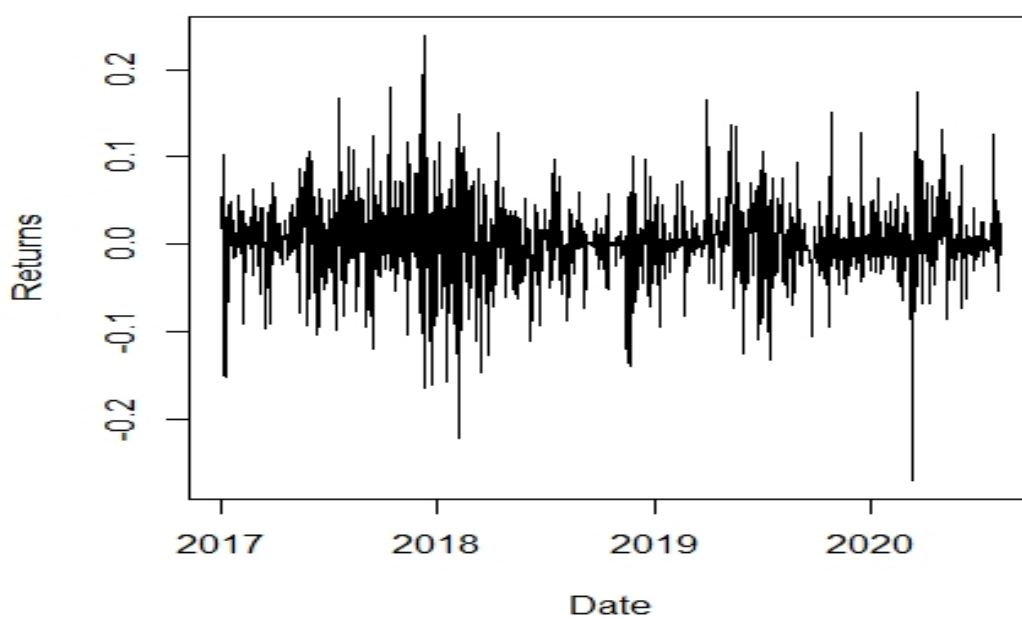


Figure 5.2: Bitcoin Return Price Plot



In order to study further return on bitcoin price, we have plotted Q-Q plot in Figure 5.3. It can be seen in following figure that the both end of the distribution is far from straight lines and it is called fat-tailed Q-Q plots. Hence it is far from normal distribution in price return. The presence of fat tail clearly implies risk associated with investment. Also, it is also believed that fat tail in bitcoin return can be attributed to investors behaviour and a topic of discussion in finance from behavioural perspective.

Next set of study was done using Auto-correlation Function to see positive or negative correlation with time. ACF is plotted between auto-correlation and lagged time. These ACF are the way to explain relation between price data and time series data. ACF plot for price data is shown in Figure 5.4 and there is positive correlation of bitcoin price with time but it is reducing over a span of time periods. ACF plot for daily return is shown in Figure 5.5 and it is quite visible that auto-correlation is below dotted line, forcing us to believe that return is not dependent on the time.

Figure 5.3: Q-Q Plot for Bitcoin Return Price

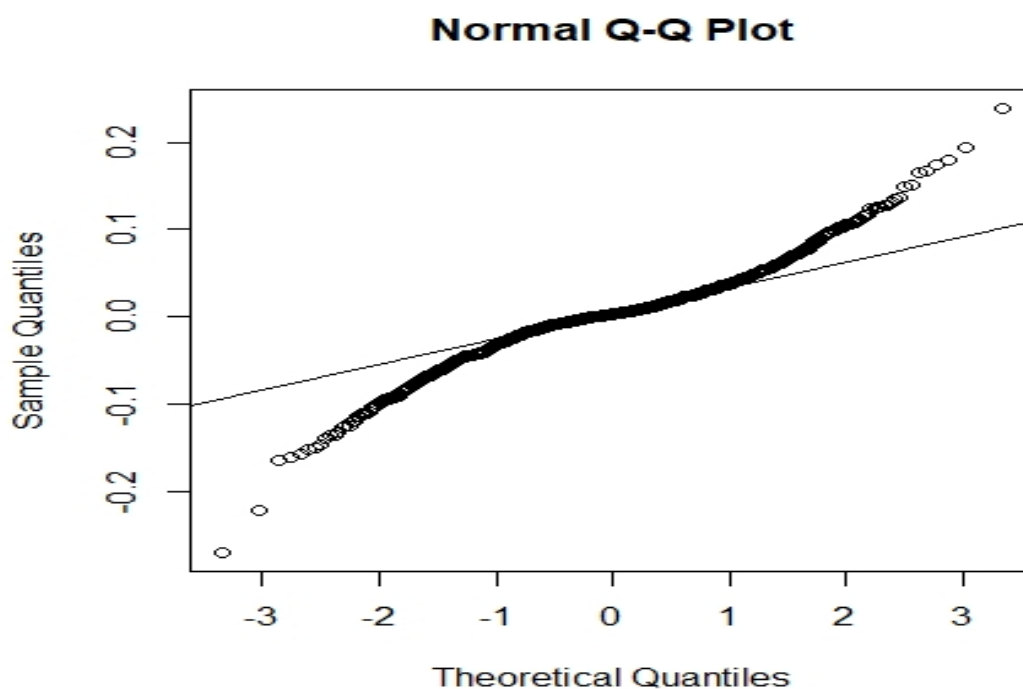


Figure 5.4: ACF Plot for Bitcoin Price

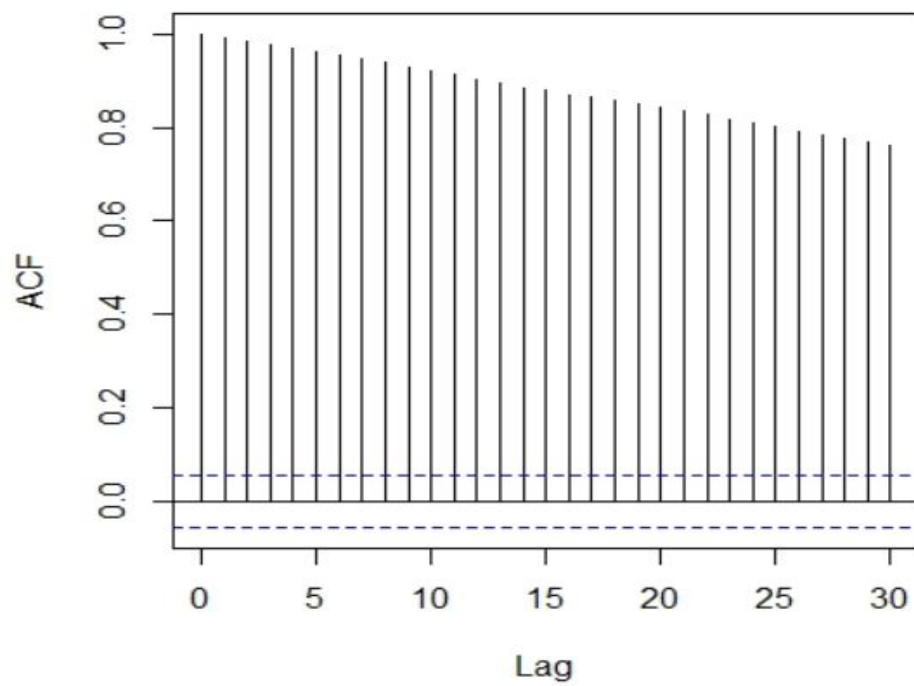
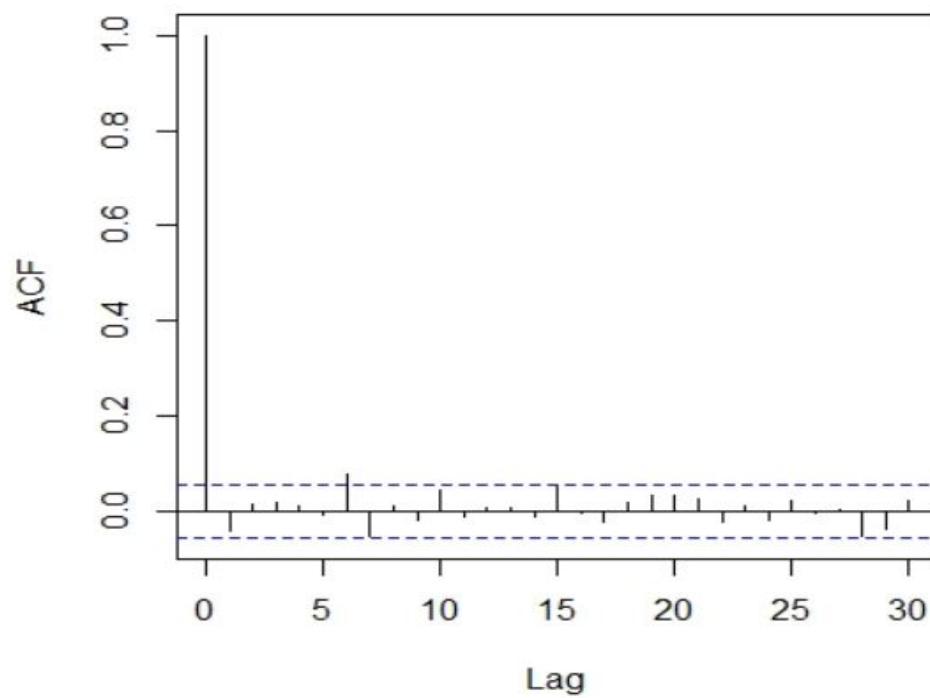


Figure 5.5: ACF Plot for Bitcoin Return Price



### 5.3 Comparison with Other Assets

For this section, we shall be comparing bitcoin return data with other assets. We picked other assets such as oil, gold, US real estate index and Standard and Poor 500 index. We got all the data from Yahoo finance as it is part of R package. In order to compare returns better, we have picked same time window from January 1, 2017 to August 10, 2020. As we know already in advance, these kinds of assets are only traded on weekdays except public holidays. We have plotted return of these assets as given in below Figure 5.6 and it is quite visible that these assets returns are very much sensitive or floating around as compared to Bitcoin. It can be deduced that Bitcoin can actually add diversification in investors portfolio along with trade-off of risk.

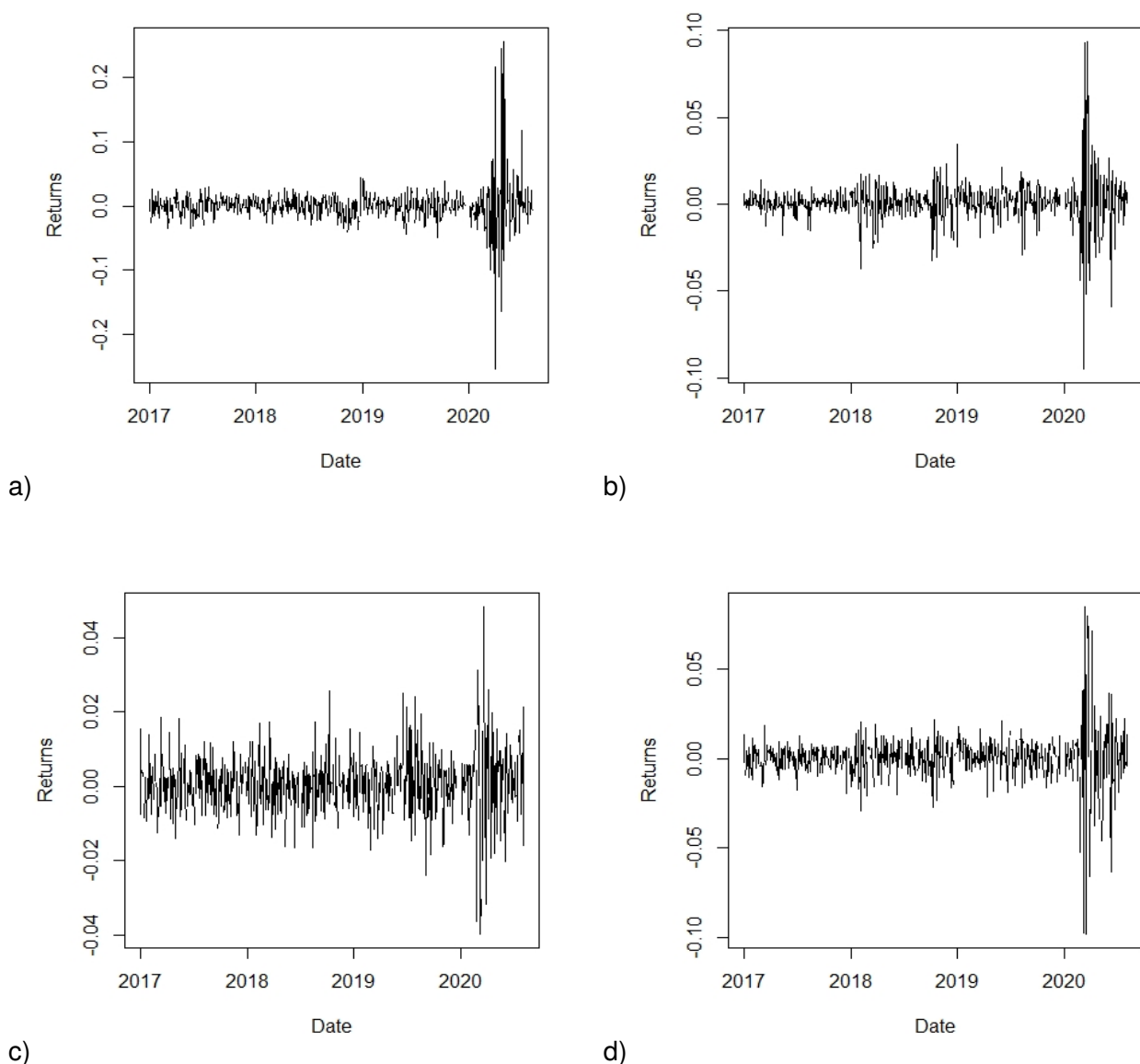


Figure 5.6: (a) Oil Return (b) SP500 Return (c) Gold Return (d) US Real State Index

## Chapter 6

# Implementation

### 6.1 Bitcoin Bubble Generation Phases

In order to analyse bubble generation phase, we have divided our windows in two time periods. These two time periods are the end of year 2017 and the beginning of year 2020. In 2017, the beginning period  $t_1$  is set to September 2, 2017 and the ending period  $t_2$  is set to December 1, 2017. To predict the price movement in 2017, we shall be using 90 days forward window. By using above methodology, we will try to predict the actual date when crash happened along with the maximum price. Then, we will compare predicted price with actual price and actual date of the bubble and crash to establish the efficacy of our model.

Similarly, we have chosen 60 days forward window to predict second bubble in the beginning of year 2020. In 2020, the beginning period  $t_1$  is set to December 10, 2019 and the ending period  $t_2$  is set to February 10, 2020. we shall be looking the bubble crash date here and later, we will compare the prediction with the actual date and price. These 60 days and 90 days windows are chosen based on accuracy of results achieved.

To implement and validate the model, CMA-ES algorithm will be used. Also, we have discussed in CMA-ES algorithm section that we will not get global optima in just one run of this model. Hence, we will fit our model 100 times and build a confidence interval to predict the price so that results look more promising. As depicted in below Figure 6.1, all curves are plotted, with the green line represents the start of period  $t_1$ , blue line represents the end of period  $t_2$  and red line represents the actual bubble crash date on December 17, 2017 and February 15 in the year 2020. Note that the price in logarithmic scale is shown on Y axis.

From Figures 6.1 and 6.2, we can observe that during this period, New JLS factor model can reproduce the Bitcoin price movement in both the years. The frequency at which oscillation moves is not changing under log scale; However, on linear scale, this frequency is gradually going up. Further,

it is observed that the movement in bitcoin price does not follow exponential growth rather it follows "faster-than-exponential growth" [20]. Earlier models such as Old JLS model assumed that bubbles and crashes are the results of interaction among only market participants, while other factors hardly influence price movement. But Zongyi Hu and Chao Li modified JLS model in order to incorporate other economic factors such as interest rate, exchange rate and historical volatility of an asset. In our area of research, we have introduced historical volatility of bitcoin price to predict its behaviours over two-time windows.

It can be seen from both the Figures 6.1 and 6.2, as the peak period  $t_c$  reaches, the change in the price of the bitcoin will keep on moving upward till the crash happens in both the years. This price movement supports the strategy "cut the losers and run the winners".

Also, this is observed that a large crash is mainly result of the imitation between participants and their herding investment strategy. This is called positive "price to- price feedback mechanism", leading to a bubble. The rise in asset price attracts customers to keep buying and waiting, and the extra risk of happening bubble. It is usually prevalent among customers that many will try to imitate their friends by selling the order at the same time, resulting in a crash.

Figure 6.1: Predicted Price for Bubble Phase: 2017

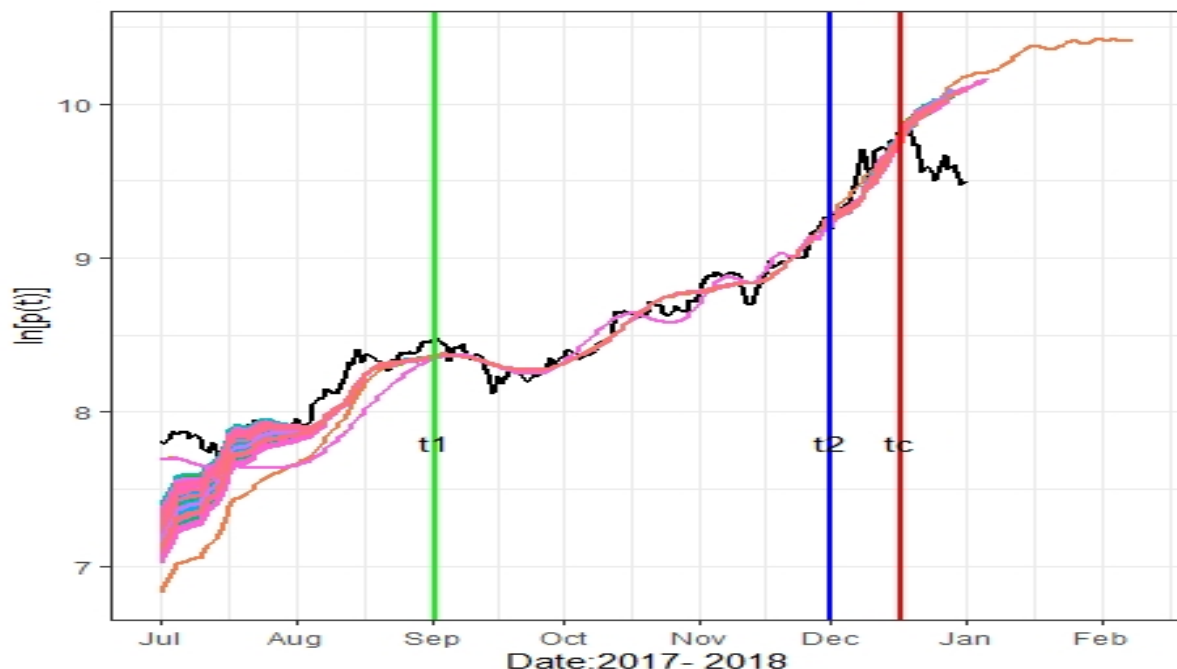
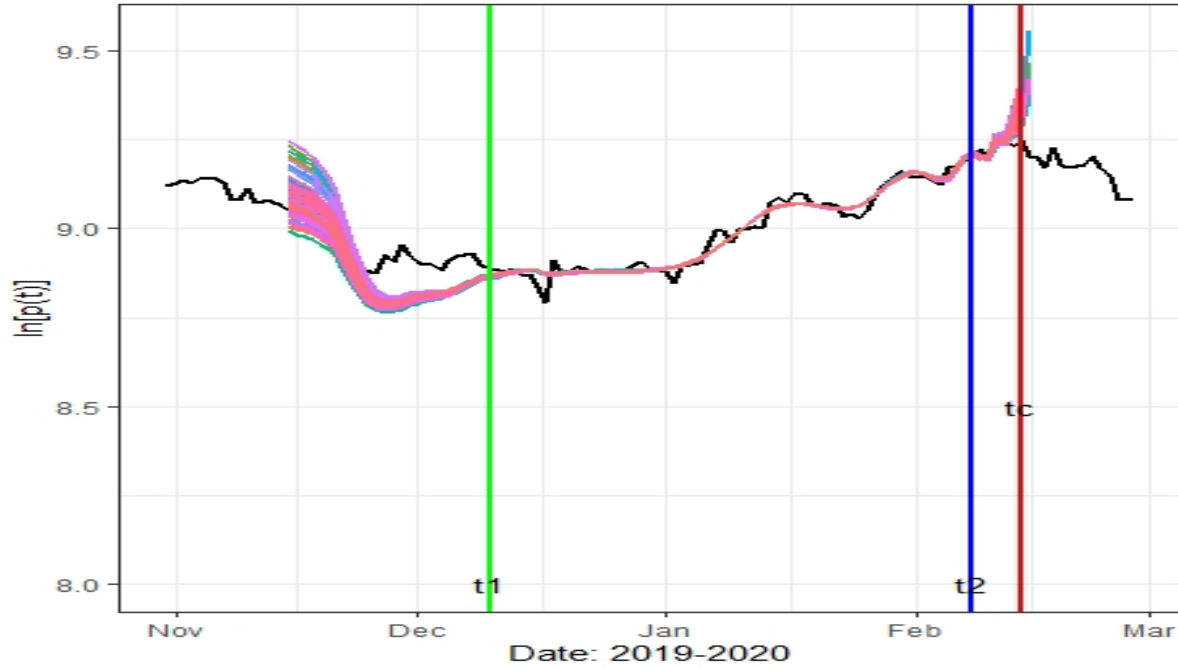


Figure 6.2: Predicted Price for Bubble Phase: 2020



## 6.2 Bitcoin Anti-Bubble Phases

For bubble crash phase in 2017, we have set the beginning period  $t_1$  to December 18, 2017 one day just after the bubble peak in 2017 and the end period  $t_2$  to be March 28, 2018. Similarly, we have set the beginning period  $t_1$  to February 15, 2020 and the ending period  $t_2$  to March 30, 2020. Here, our main focus is to fit our New JLS factor model and validate if our model can make good prediction during crash phase as well. Note that we already know about peak point  $t_c$ , hence the number of non-linear parameters reduces from 3 to 2, reducing the model complexity in a significant manner. Also, our main equation has been changed to below equation for crash phase since it produces better result with less parameters [20].

$$\ln[p(t)] = A + B|t - t_c|^\beta + C|t - t_c|^\beta \cos(\omega \ln|t - t_c| + \phi) \quad (6.1)$$

As we did for bubble generation phase, we fit the CMA-ES algorithm 100 times and plot all the curves as given in Figure 6.3 and 6.4. The red line shows the starting period  $t_1$  and the green line shows the end of the period  $t_2$ . From the graph, it is quite evident that model predicted price are matching with actual prices. Since results are converging, we can trust the estimated parameters along with model prediction.

Figure 6.3: Predicted Price for Crash Phase: 2017

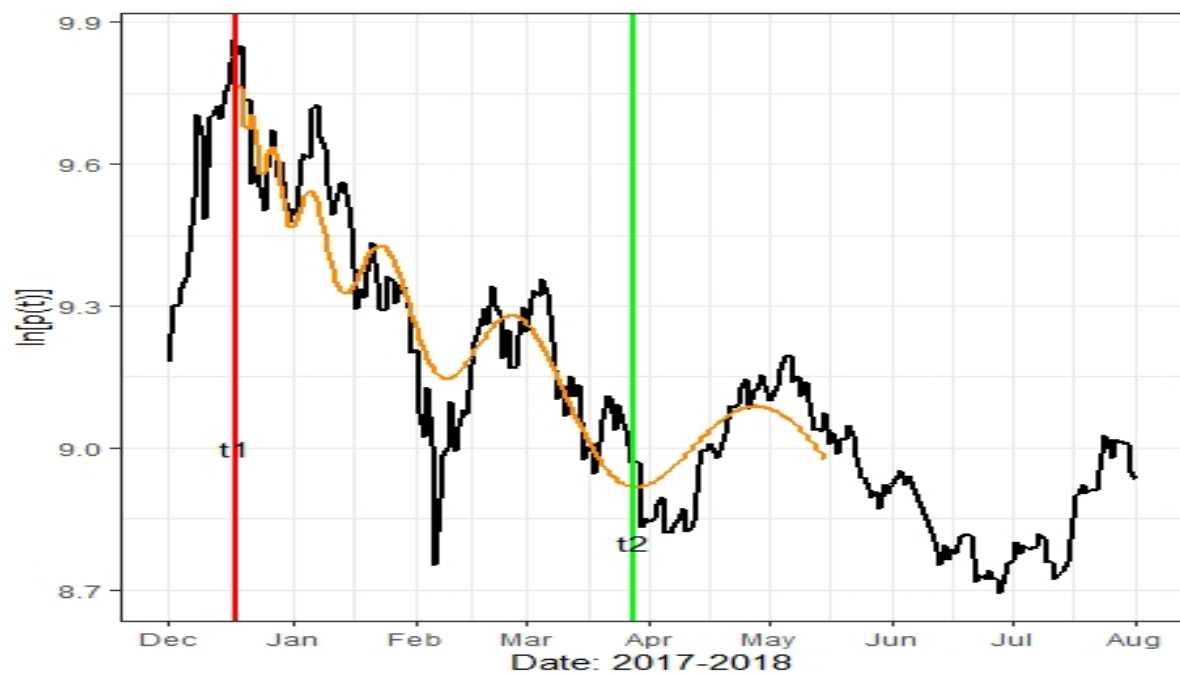
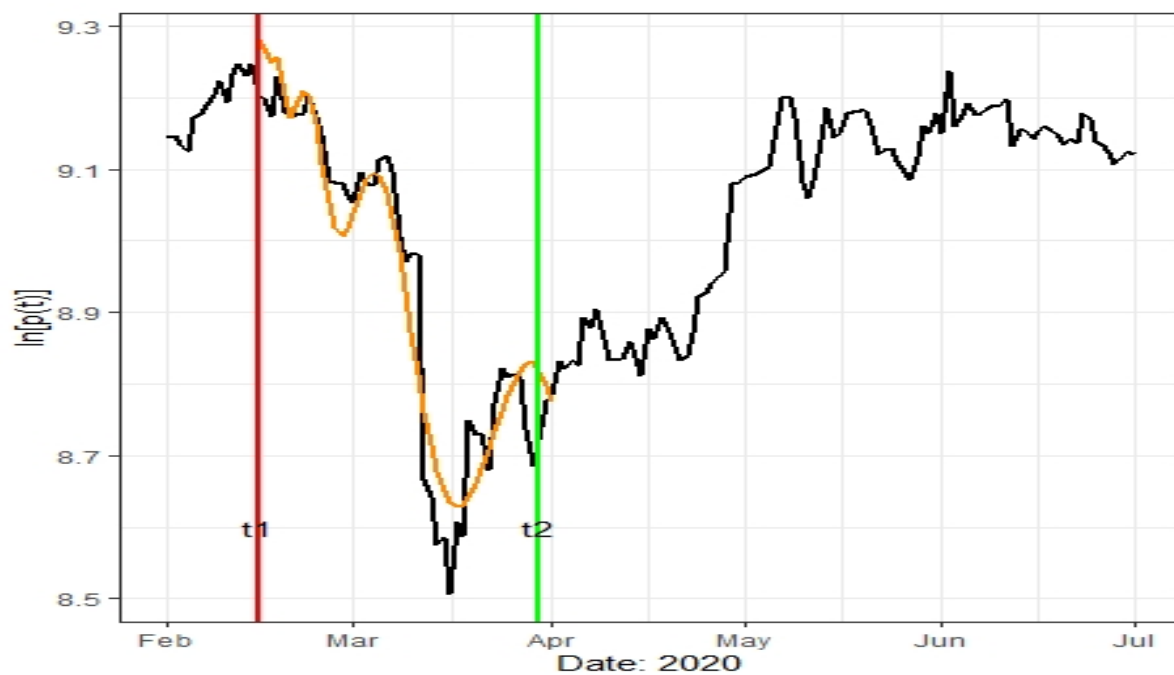


Figure 6.4: Predicted Price for Crash Phase: 2020



### 6.3 Future Price Prediction

In this section of bitcoin price prediction, we will try to simulate prediction on recent data starting from March 2020 to August 2020. We have observed in previous sample data that there was another bubble post December, 2017 that occurred on February 15, 2020 when bitcoin price reached at \$10,364 and there was sudden crash with minimum price reached at \$4,944 on March 16, 2020, an almost half the price. We can see that prices started spiralling again and reached to maximum at \$11,673 on August 10, 2020 till the time we have collected data for our study. Are we going to experience another bubble any time soon in upcoming months and then sudden crash? We can verify this by using our New JLS factor model implemented and validated on previous bitcoin price data in December, 2017 and February, 2020. We have chosen two time windows specific to 60 days and 90 days forward rolling window. Since we only have data available until August 10, 2020, we will restrict our end of period to August 10, 2020. The start period  $t_1$  was set to June 10, 2020 for 60 days forward rolling windows and the end period  $t_2$  was set to August 10, 2020. Similarly, the start period  $t_1$  was set to May 10, 2020 for 90 days forward rolling windows and the end period  $t_2$  was set to August 10, 2020. After fitting the data in New JLS factor model with different time periods, the predicted price curve and the histogram of days to peak are shown in Figure 6.5 and Figure 6.6, correspondingly.

From the below Figures 6.5 and 6.6, it can be seen easily that New JLS factor model can fit the price change over 2 different time windows. It is also observed that the price movement is of quite similar frequency at log scale. As for the days till peak window as given in Figures 6.7 and 6.8, the predicted critical time  $t_c$  will happen in 25 days post August 10, 2020 with 60-days forward window, while the predicted critical time or another bubble can be expected to happen in 34 days after August 10, 2020 with 90 days forward window. With 60 days forward window, the expected time of another bubble may happen by September 5, 2020 with maximum price of \$31232.17 since bitcoin price is surging day by day in the month of August 2020. Similarly, with 90 days forward window, it can be expected to reach \$35320.120 by September 14, 2020. We can use any window to predict future price movement for bitcoin. As evident from the price movement, with two different forward windows, the peak price of bitcoin in future is beyond estimation though we have included exogenous factor such as historical volatility of bitcoin price data.

Figure 6.5: Predicted Price by 60 days rolling windows: 2020

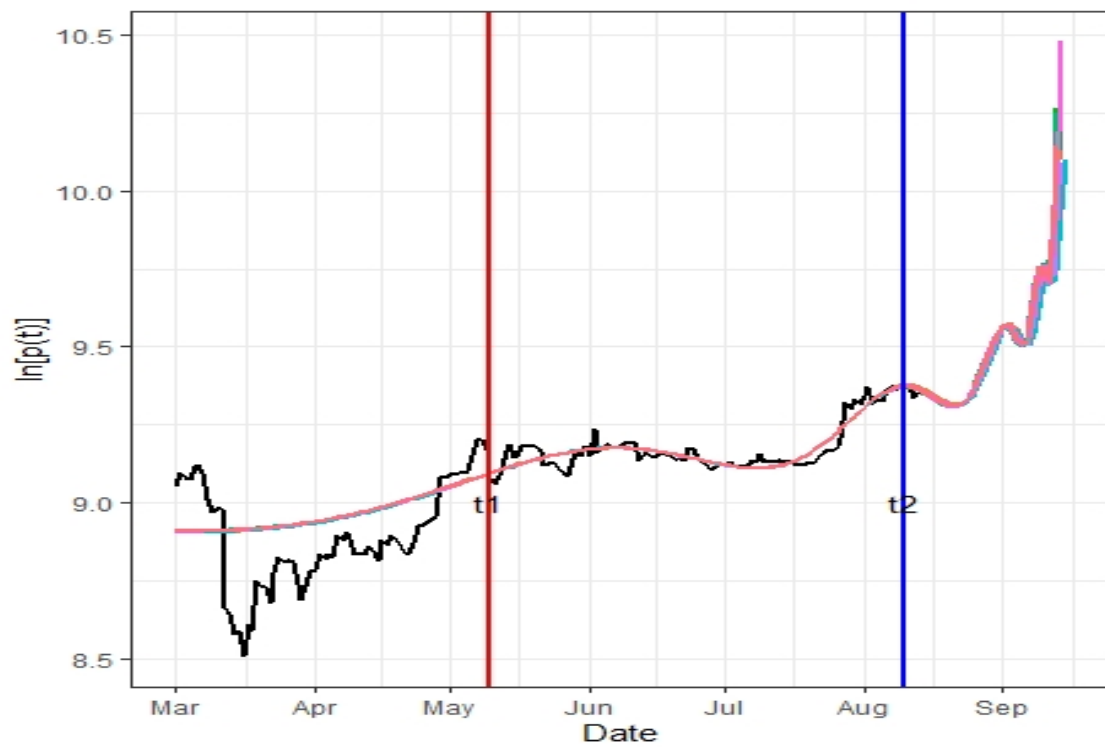


Figure 6.6: Predicted Price by 90 days rolling windows: 2020

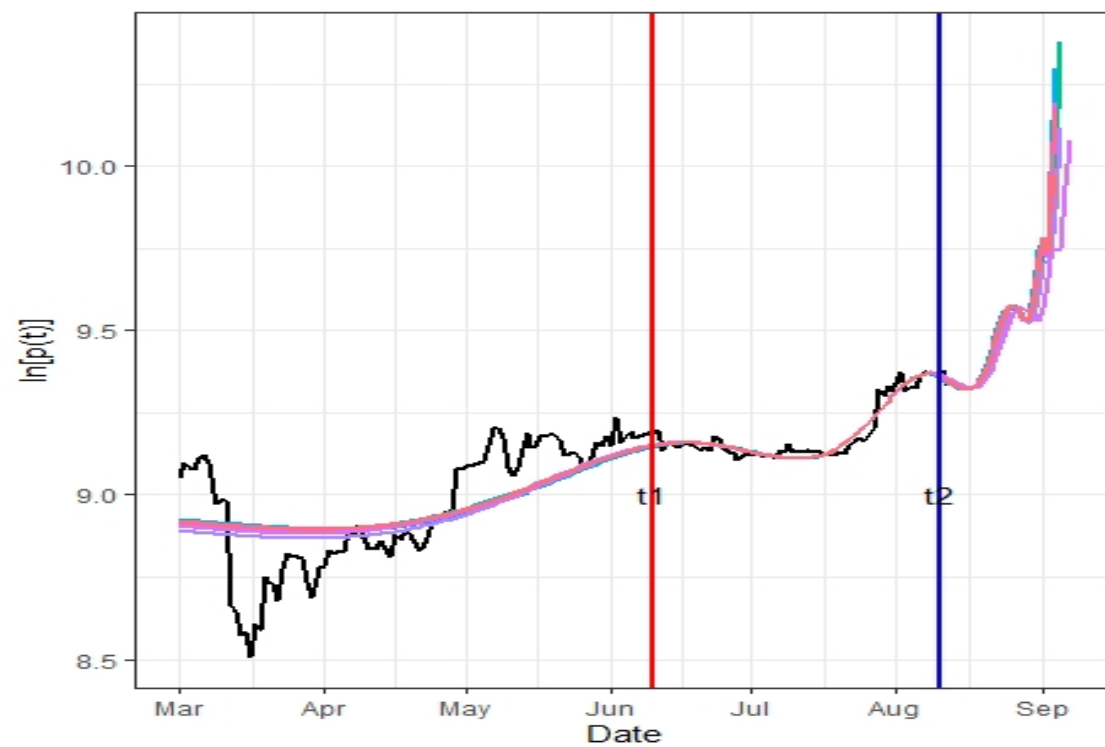


Figure 6.7: Number of days to reach peak in 2020: 60 days window

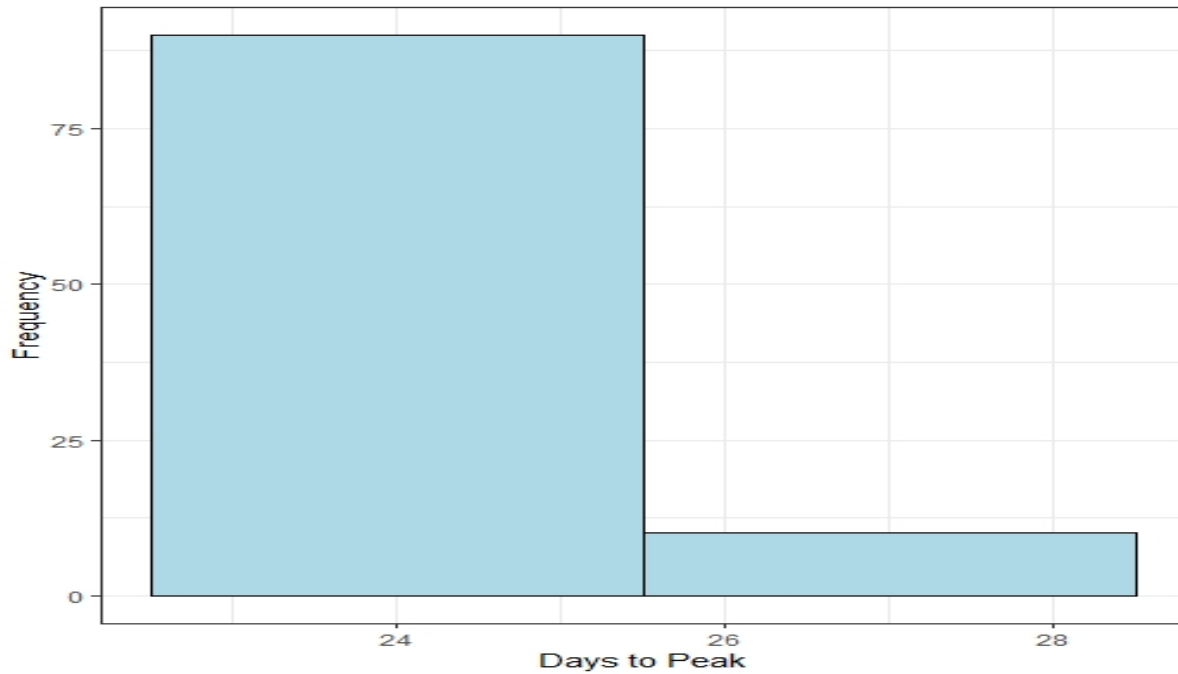
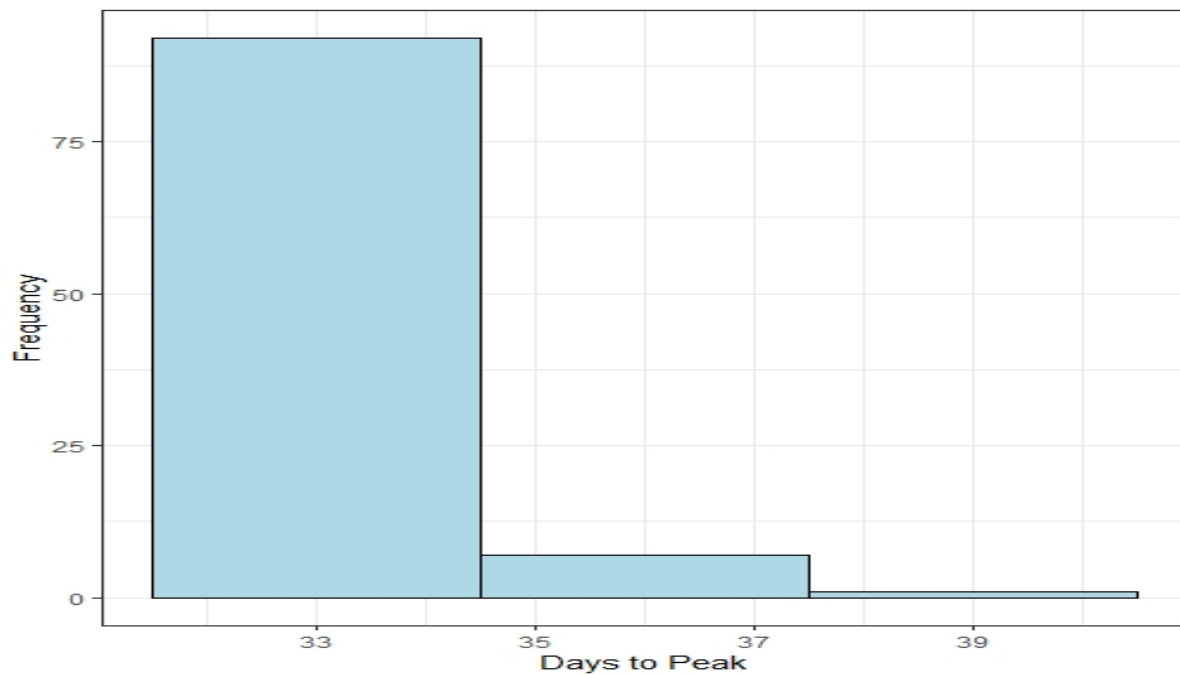


Figure 6.8: Number of days to reach peak in 2020: 90 days window



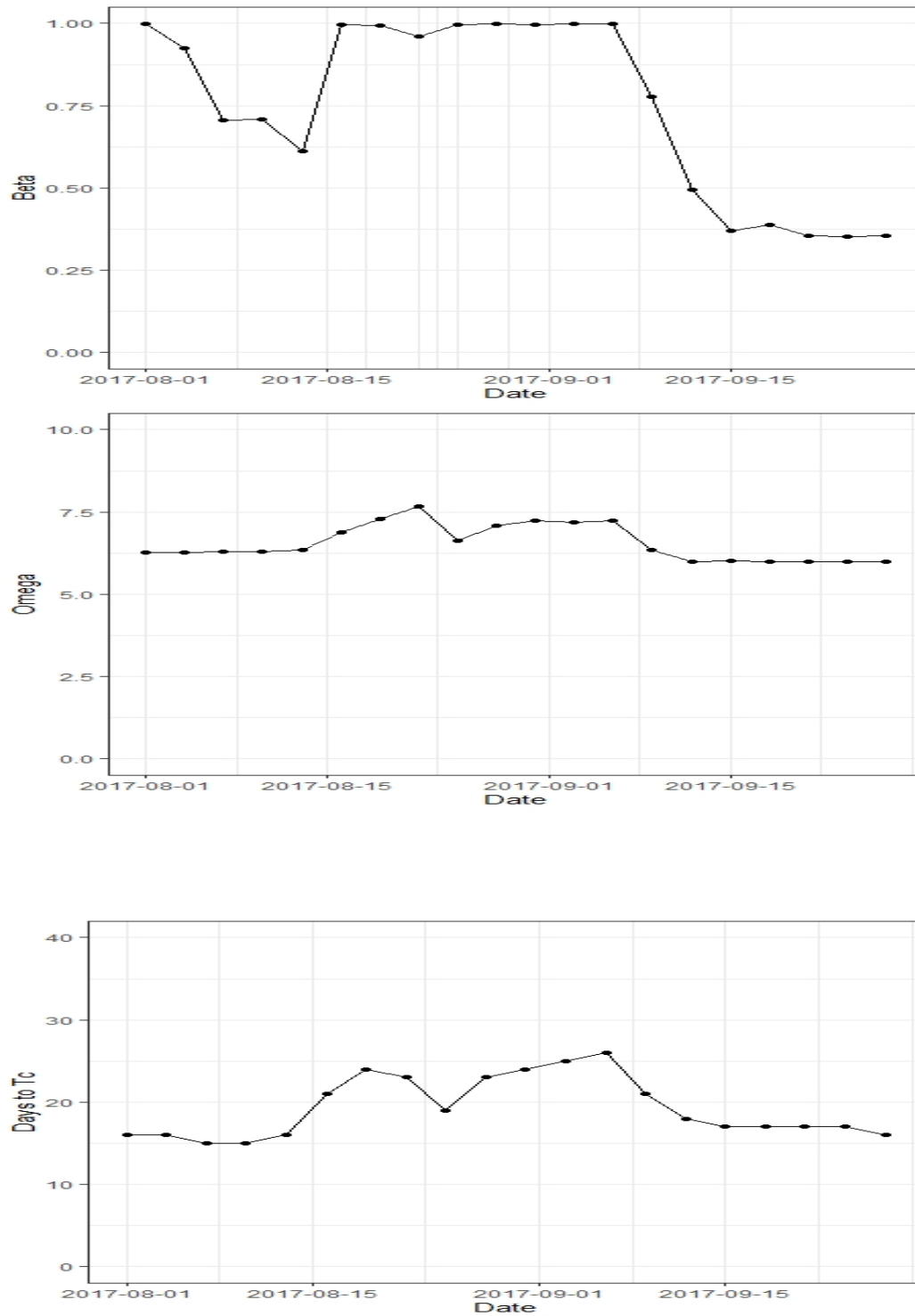
## 6.4 Market Sensitivity Parameter Analysis

This specific section will focus on non-linear parameters ( $\omega$ ,  $\beta$ , Days to peak) analysis. The whole idea is to perform market sensitivity analysis on the bubble phase in 2017. We shall not be conducting this analysis on bubble crash phase since result which we got are quite converging and this analysis is very specific to bubble phase. It becomes quite difficult to do the same analysis on price prediction for future dates since we relied on existing data along with predicted price for a given window. Hence, our validation can be more accurate and authentic for the time periods for which we have bitcoin price data available. The step by step process to perform market analysis is as follows: firstly, we shall be picking two time windows as we did earlier for bubble phase and prediction phase. Since the day for bubble occurring is fixed, we will use forward window approach and keep on modifying starting window. We will be fixing our end period window  $t_2$  on December, 2017. However, initially we will keep our starting time window  $t_1$  to August, 2017. We will reset our time window starting from  $t_1$  by every third trading days,  $\Delta t=3$ .

Now we will fit our New JLS factor model by using CMA-ES algorithm 50 times. These steps will be executed 20 times in order to vary the forward window from 60 days to 120 days since price prediction is always dependent on time windows and we need to be very cautious about forward time window for analysis. We will get value of each parameters for every 20 starting dates. As we already know in advance that the calculation for linear parameters will be done based on non-linear parameters. Hence we will be plotting these parameters in below sub Figures 6.9.

As it is quite evident from below Figure 6.9(middle) that the value of  $\omega$  is quite stable for initial 15 days and then changes as forward window increases. However the value of  $\beta$  is varying between 0.4 and 0.99. Also, its value was quite stable for a specific time window from August 15, 2017 to September 9, 2017. We focus more on peak time period since it is our most important parameter which will validate our over-all prediction. The value of days to peak changes from 15 to 25 days but becomes quite stable to 15 days as the time windows approaches toward bubble phase. By looking at the above results, we can deduced that our New JLS factor model prediction are quite convincing and our model can be extended further to analyse other stock price pattern as well in future.

Figure 6.9: Market Sensitivity Parameter Analysis



## Chapter 7

# Results

### 7.1 Bubble Phase Prediction

There are almost eight parameters to be calculated, but we focused the most the two parameters: the peak point  $t_c$  and the asset price at critical point in both the years. From Table 7.1 and 7.2, it can be seen easily, when the end of period is set on December 1, 2017, the number of days to reach peak is [16,35] under 95% confidence interval in 2017, so the respective date of bubble starts from December 16 to December 30 (See the histogram of days to critical date in Figure 7.1). As a matter of fact, the peak price reached at noon on December 17, at an unprecedented \$19,521 per coin. From here, we can deduce that NJLS factor model gives prediction near to the critical time  $t_c$  with confidence interval of 95%. Also for the peak price, NJLS model gives the 95% confidence interval from \$12,510.40 to \$23,642.69 per coin, which is much higher than the actual level and predicted price touched \$18353.976 on December 17, 2017.

Similarly, we focused on critical point  $t_c$  and the asset price at peak in 2020. The end of period is set on February 10, 2020, the number of days to reach peak is [4,5] under 95% confidence interval. According to the results, the respective estimated date starts from February 15 (please check the histograms in Figure 7.2). We also know from the actual peak price reached on February 14, at \$10,367.53 per coin. From above observation, it can be deduced that NJLS model provides the prediction for peak price from \$9358.064 to \$12089.269, which is little higher than the actual level but quite closer to the peak price in the month of February, 2020. Overall, it can be derived that NJLS model gives better prediction for Bitcoin bubble in 2020 by using 60 days forward rolling window.

Table 7.1: Parameters calculated from New JLS model: Bubble Phase-2017

Parameters	Mean	Median	Standard Deviation	Confidence Interval(90%)	CI(95%)
Days to Peak	24	23	6.72	[17,34]	[16, 35]
A	7.49	7.46	0.19	[7.29, 7.68]	[7.29, 7.75]
B	-0.70	-0.66	0.29	[-1.06, -0.34]	[-1.19, -0.33]
C	0.68	0.69	0.097	[0.55, 0.80]	[0.47, 0.81]
$\beta$	0.99	0.998	0.0359	[0.989, 0.999]	[0.986, 0.999]
$\omega$	7.27	7.04	1.15	[6.18, 8.75]	[6.12, 9.73]
$\phi$	0.613	0.50	0.473	[0.053, 1.345]	[0.042, 1.64]
$\varphi$	4.513	4.56	0.425	[4.19, 4.90]	[4.15, 4.92]

Table 7.2: Parameters calculated from New JLS model: Bubble Phase-2020

Parameters	Mean	Median	Standard Deviation	Confidence Interval(90%)	CI(95%)
Days to Peak	4	4	0.288	[4,5]	[4,5]
A	10.2	10.1	0.29	[10.01, 10.54]	[9.98, 11.04]
B	-1.38	-1.323	0.20	[-1.53, -1.30]	[-1.92, -1.30]
C	0.073	0.075	0.009	[0.056, 0.085]	[0.048, 0.088]
$\beta$	0.31	0.33	0.06	[0.212, 0.380]	[0.144, 0.391]
$\omega$	8.51	8.45	0.19	[8.32, 8.94]	[8.31, 9.03]
$\phi$	0.04	0.043	0.232	[0.039, 0.052]	[0.039, 0.054]
$\varphi$	-5.7	-5.5	0.544	[-6.66, -5.00]	[-7.07, -4.96]

Figure 7.1: Number of days to reach peak in Bubble Phase: 2017

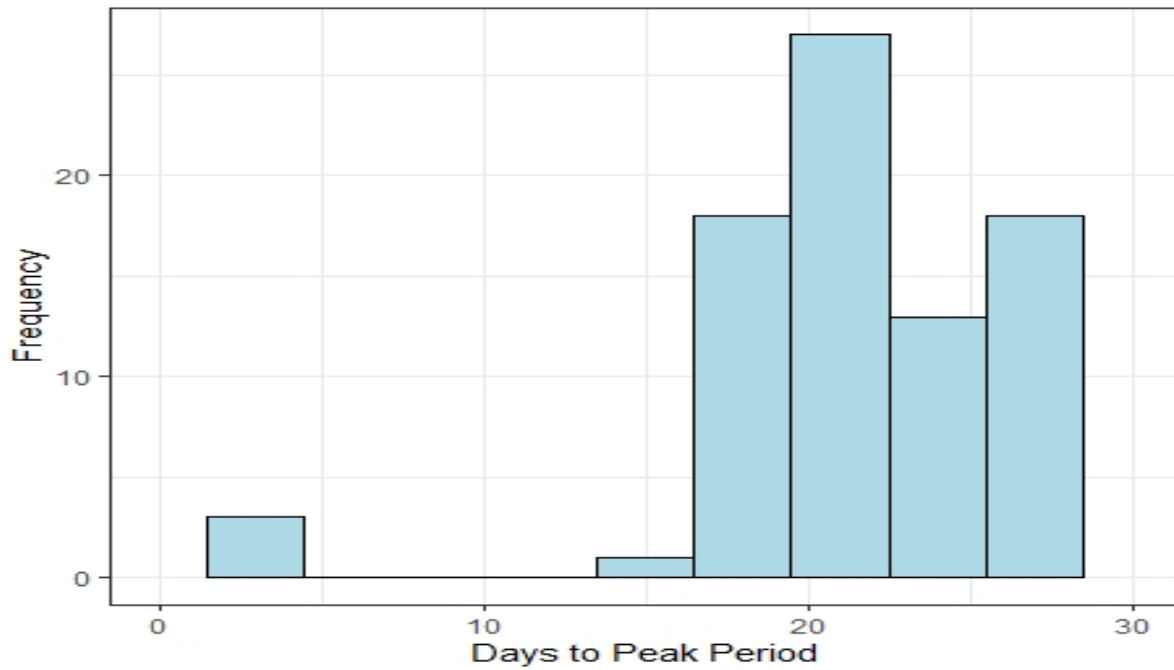
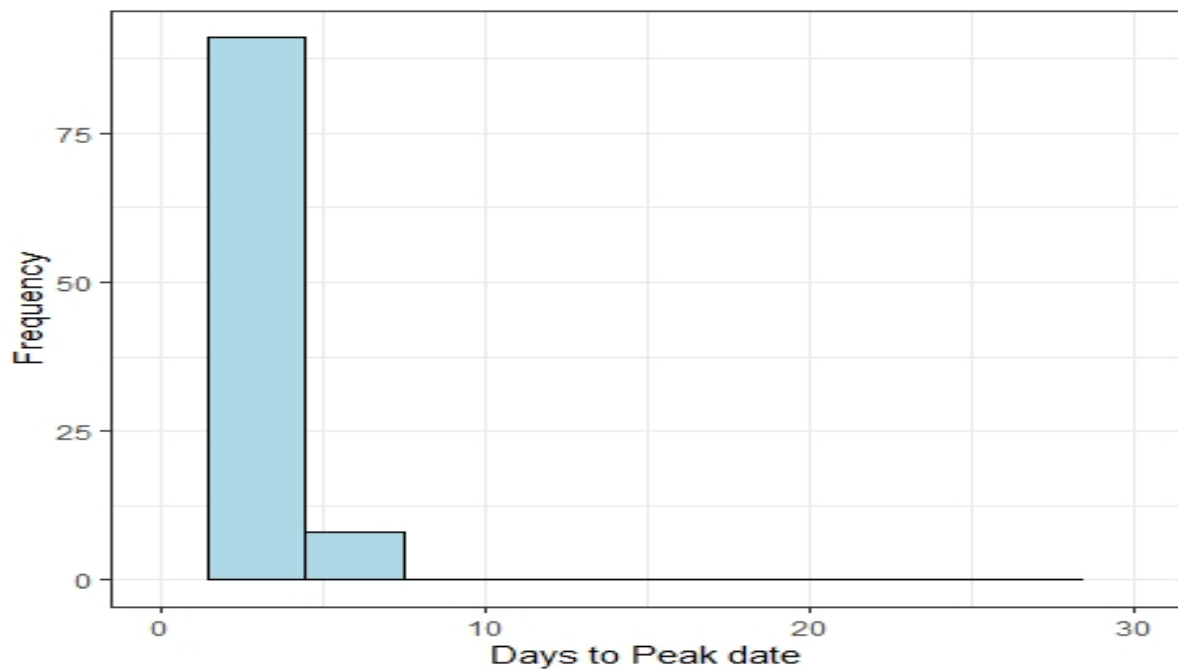


Figure 7.2: Number of days to reach peak in Bubble Phase: 2020



## 7.2 Anti-Bubble Prediction

The value of parameters is shown in Table 7.3 and 7.4 for both the years 2017 and 2020. It can be observed that, the frequency of oscillation is much higher than that we had it in the bubble phase, represented by the higher omega value. Also, New JLS model is perfectly illustrating the price movement over the time period, resulting the price change to be in line with the anti-bubble phase. During crash phase of 2017, the Bitcoin price is following log-periodic pattern. It is also reported from the price prediction that the accuracy of price prediction over the actual price is not better than the accuracy in bubble phase. As it can be seen from the analysis from the tables for year 2017, New JLS model predicts lowest price to be \$9,509.61 per coin which is at least \$3,000 higher than actual price on February 2018. One parameter is reduced in anti-bubble phase and the explanation is already provided in section 6.2.

However, the prediction for the year 2020, anti-bubble period, the lowest price happened on March 16, 2020 at \$4,944.70 which is \$600 higher than predicted price \$5,620.671 per coin. The prediction for anti-bubble period looks better than prediction in year 2017 anti-bubble period. This can be attributed to different time periods and the complexity of the model and there is scope for improvement in our New JLS factor model.

Table 7.3: Parameters for New JLS Factor Model: Crash Phase in 2017

A	B	C	$\omega$	$\beta$	$\phi$
9.957	-1.516	0.220804	10.05	0.3982	0.04

Table 7.4: Parameters for New JLS Factor Model: Crash Phase in 2020

A	B	C	$\omega$	$\beta$	$\phi$
9.3	-5.867	2.03	7.562	1	0.05

## 7.3 Price Prediction: Another Bubble in 2020?

The estimated parameters are given in below Tables 7.5 and 7.6 for both 60 days and 90 days rolling window. When we explored post bubble analysis as in Section 6.1 and 6.2, since we have already known the critical time, it is quite easy to do the reverse analysis to verify the prediction and date

predicted. However, it does not give us the actual event such as whether bubble predicted would be local or global one. In a nutshell, we can say that New JLS factor model can capture the price change really well so far on bitcoin price, there might be a possibility that future bubble might happen a little early or a little late in September, 2020 as we only have predicted date for next bubble date with 95% confidence interval. Overall, the price of an asset represents interaction among market factors and only future price of bitcoin will tell if our model can be the preferred choice for future price prediction over Old JLS model or LPPL model.

Table 7.5: Parameters for another Bubble 2020: 60 days window

Parameters	Mean	Median	Standard Deviation	Confidence Interval(90%)	CI(95%)
Days to Peak	25	25	0.55	[25, 26]	[24, 27]
A	26.2	28.54	4.71	[15.98, 29.37]	[12.46, 29.39]
B	-17.41	-19.76	4.69	[-20.59, -7.20]	[-20.60, -3.71]
C	0.071	0.069	0.004	[0.0696, 0.073]	[0.069, 0.079]
$\beta$	0.017	0.010	0.023	[0.010, 0.030]	[0.010, 0.067]
$\omega$	6.015	6.001	0.047	[6.00, 6.07]	[6.0, 6.12]
$\phi$	0.042	0.042	0.062	[0.040, 0.043]	[0.040, 0.0435]
$\varphi$	6.2	6.34	0.034	[5.25, 6.55]	[5.01, 6.77]

Table 7.6: Parameters for another Bubble 2020: 90 days window

Parameters	Mean	Median	Standard Deviation	Confidence Interval(90%)	CI(95%)
Days to Peak	34	34	0.72	[34, 35]	[34.000, 35.53]
A	26.35	27.64	2.9	[18.75, 28.19]	[17.75, 28.19]
B	-17.5	-18.8	2.91	[-19.3, -9.9]	[-19.34, -8.92]
C	0.077	0.077	0.0006	[0.077, 0.0779]	[0.0765, 0.0788]
$\beta$	0.012	0.010	0.003	[0.010, 0.019]	[0.010, 0.024]
$\omega$	6	6	0.015	[6.00, 6.03]	[6.00, 6.053]
$\phi$	0.020	0.020	0.047	[0.019, 0.0205]	[0.0187, 0.02159]
$\varphi$	6.7	6.76	0.24	[6.5, 7.01]	[6.06, 7.24]

## Chapter 8

# Conclusion

In this research paper, we firstly discussed Bitcoin bubble, crash and the life cycle of the bubble. Bitcoin price has been very volatile since its beginning and we also know that its price had reached unprecedented price of \$19,166 in the year 2017 and then a crash followed. According to many economists, such behaviour of bitcoin price movement can be attributed to participants herding behaviour in which everyone does the same activity such as buying and selling at the same time [20]. For this reason, models such as JLS (LPPL) were able to predict price movement earlier. However, Bitcoin price movement can also be attributed to other fundamental factors such as volatility of bitcoin price over the years. For this reason, we have used New JLS factor model.

We have also used New JLS factor model to study Bitcoin price movement such as Bitcoin bubble, crash and price prediction in the year 2020. We predicted the date of bubble happening closer to actual date of bubble with deviation in 2017 as well as in 2020 with 95% confidence interval. Similarly, the prediction from the model during crash phase has also followed the actual pattern of bitcoin crash though we have reduced parameters to be used in the equation for the anti-bubble prediction.

We have picked another 60-days and 90-days forward rolling window in the year 2020 in order to predict future price of bitcoin in the year 2020. We might expect another bubble in the mid-September, 2020 as per the model prediction though it can be local or global bubble since bitcoin prices are spiralling since August, 2020.

Later, we have done analysis on model parameters using market sensitivity analysis by keep on changing the starting window and results are quite stable except for specific time windows, indicating the accuracy of model quite convincing.

Overall, it can be concluded that New JLS factor model gives nearly accurate prediction for bitcoin price movement with the fact that market fundamental factors also play an important role for Bitcoin price prediction. Also, we can conclude that Bitcoin bubbles and crashes behaviour can also be studied by using Statistical Models (New JLS factor model) rather than complex Neural Networks.

## Chapter 9

# Future Work

Though extended version of JLS model(LPPL) was used in my thesis and our model provide estimation of peak time of bubble and crash near to actual date of bubble, the results are obtained with 95% confidence interval and there is deviation with actual date of bubble and the model can be improved further to be used as a generic model for other stocks as given below:

- The model can be enhanced further by introducing more market fundamental factors such as interest rate, deposit reserve rate, interest spread, implied volatility, and exchange rates and the enhanced model can be used to study other stock market bubbles and price movement.
- As discussed in demonstration with Professor, we can calculate historical volatility using interpolation method described in the book (An Introduction to High-Frequency Finance) [16]. By calculating volatility using below formula, we can enhance our model to give prediction in shorter time window and bitcoin price movement can be tracked on daily basis. This method can be employed to perform further research on my thesis and the prediction near to actual time of Bubble can be made.

$$v(t_i) = v(\Delta t, n, p; t_i) = \left[ \frac{1}{n} \sum_{j=1}^n |r(\Delta t; t_{i-n+j})|^p \right]^{1/p} \quad (9.1)$$

where  $r(\Delta t; t_i) = x(t_i) - x(t_i - \Delta t)$ .  $x(t_i)$  is called log price,  $r(t_i)$  is the return and  $v(t_i)$  is the volatility.

Also scaled volatility can be calculated by below formula.

$$v_{scaled} = \sqrt{\frac{\Delta t_{scale}}{\Delta t}} v \quad (9.2)$$

By making above changes to the model, we can further broaden our scope of study by applying different time windows and can make more accurate prediction about Bitcoin bubble and crash in the future.

# Bibliography

- [1] The Guardian (2018). "Bitcoin Biggest Bubble in History, says economist who predicted 2008 crash" [online] available at <https://www.theguardian.com/technology/2018/feb/02/bitcoin-biggest-bubble-in-history-says-economist-who-predicted-2008-crash>. 2018.
- [2] Aamna Al Shehhi, Mayada Oudah, and Zeyar Aung. "Investigating Factors Behind Choosing a Cryptocurrency". In *2014 IEEE international conference on industrial engineering and engineering management*, pages 1443–1447. IEEE, 2014.
- [3] Robert Z Aliber and Charles P Kindleberger. *"Manias, panics, and crashes: A history of financial crises"*. Springer, 2017.
- [4] Saifedean Ammous. *"The Bitcoin Standard: the Decentralized Alternative to Central Banking"*. John Wiley & Sons, 2018.
- [5] Andreas M Antonopoulos. "Mastering Bitcoin 2nd Edition. Giugno 2017".
- [6] Adam Back et al. "Hashcash-a denial of service counter-measure". 2002.
- [7] Claude Comtois and Brian Slack. *"The geography of transport systems"*. Routledge, 2009.
- [8] Anne Haubo Dyhrberg. "Hedging Capabilities of Bitcoin. Is it the virtual gold?". *Finance Research Letters*, 16:139–144, 2016.
- [9] John Eatwell, Murray Milgate, Peter Newman, and Peter K Newman. *"The new palgrave: Economic development"*. WW Norton & Company, 1989.
- [10] Hermann Elendner, Simon Trimborn, Bobby Ong, and Teik Ming Lee. "The cross-section of crypto-currencies as financial assets: Investing in crypto-currencies beyond bitcoin". In *Handbook of Blockchain, Digital Finance, and Inclusion, Volume 1*, pages 145–173. Elsevier, 2018.
- [11] Vladimir Filimonov and Didier Sornette. "A stable and robust calibration scheme of the Log-Periodic Power Law Model". *Physica A: Statistical Mechanics and its Applications*, 392(17):3698–3707, 2013.

- [12] Kristin M Finklea. "Dark Web", 2015.
- [13] Sean Foley, Jonathan R Karlsen, and Tālis J Putniņš. "Sex, Drugs, and Bitcoin: How much illegal activity is financed through cryptocurrencies?". *The Review of Financial Studies*, 32(5):1798–1853, 2019.
- [14] Dr. Vassilios G. Papavassiliou. "Bitcoin Digital Currency as An Investment Asset" [online] available at <https://internationalbanker.com/brokerage/bitcoin-digital-currency-as-an-investment-asset/>. Dec 2019.
- [15] Peter M Garber. "Famous First Bubbles: the fundamentals of early manias", 2000.
- [16] Ramazan Gençay, Michel Dacorogna, Ulrich A Muller, Olivier Pictet, and Richard Olsen. *"An Introduction to High-Frequency Finance"*. Elsevier, 2001.
- [17] Petr Geraskin and Dean Fantazzini. "Everything you always wanted to know about log-periodic power laws for bubble modeling but were afraid to ask". *The European Journal of Finance*, 19(5):366–391, 2013.
- [18] David Ha. "A Visual Guide to Evolution Strategies". *blog.otoro.net*, 2017.
- [19] Nikolaus Hansen. "The CMA Evolution Strategy: A Tutorial". *arXiv preprint arXiv:1604.00772*, 2016.
- [20] Zongyi Hu and Chao Li. "New JLS-Factor Model versus the Standard JLS Model: A Case Study on Chinese Stock Bubbles". *Discrete Dynamics in Nature and Society*, 2017.
- [21] Anders Johansen and Didier Sornette. "Modeling the Stock Market Prior to Large Crashes". *The European Physical Journal B-Condensed Matter and Complex Systems*, 9(1):167–174, 1999.
- [22] Yasar Kaya. "Analysis of Cryptocurrency Market and Drivers of the Bitcoin Price": Understanding the price drivers of Bitcoin under speculative environment", 2018.
- [23] John Maynard Keynes. "Treatise on money: Pure theory of money Vol. I", 1930.
- [24] Tony Klein, Hien Pham Thu, and Thomas Walther. "Bitcoin is not the New Gold—A comparison of volatility, correlation, and portfolio performance". *International Review of Financial Analysis*, 59:105–116, 2018.
- [25] Dimitrios Koutmos. "Market Risk and Bitcoin Returns". *Annals of Operations Research*, pages 1–25, 2019.

- [26] Rohit Kukreja. "What is Bitcoin?" [online] available at <https://kryptomoney.com/advantages-and-disadvantages-of-bitcoins/>. 2017.
- [27] Alec MacDonell. "Popping the Bitcoin bubble: An application of log-periodic power law modeling to digital currency". *University of Notre Dame working paper*, pages 1–33, 2014.
- [28] Michael Mainelli and Mike Smith. "Sharing ledgers for sharing economies: an exploration of mutual distributed ledgers (aka blockchain technology)". *Journal of Financial Perspectives*, 3(3), 2015.
- [29] Satoshi Nakamoto. "Bitcoin: A Peer-to-Peer Electronic Cash System". Technical report, Manubot, 2019.
- [30] Andrew Phillip, Jennifer SK Chan, and Shelton Peiris. "A new look at Cryptocurrencies". *Economics Letters*, 163:6–9, 2018.
- [31] Emmanouil Platanakis and Andrew Urquhart. "Should Investors include Bitcoin in their portfolios? A portfolio theory approach". *The British accounting review*, 52(4):100837, 2020.
- [32] J Barkley Rosser. "*From Catastrophe to Chaos: A General Theory of Economic Discontinuities: Mathematics, Microeconomics and Finance*", volume 1. Springer Science & Business Media, 2000.
- [33] Tim Salimans, Jonathan Ho, Xi Chen, Szymon Sidor, and Ilya Sutskever. "Evolution Strategies as a scalable alternative to Reinforcement Learning". *arXiv preprint arXiv:1703.03864*, 2017.
- [34] Jeremy J Siegel. "What is an Asset Price Bubble? An operational definition". *European financial management*, 9(1):11–24, 2003.
- [35] Basil S Yamey, Richard L Sandor, and Brian Hindley. "*How commodity futures markets work*". Number 42. Trade policy research centre London, 1985.

## Appendix A

# My Github Repository Link for Implementation

All my code written in R language is available in below github directory. Please refer to below link.

<https://github.com/singh0021/Master-DataScience-Thesis>